Reinforced Concrete (RC) Structures

Topic 15.

Intermediate state of stress

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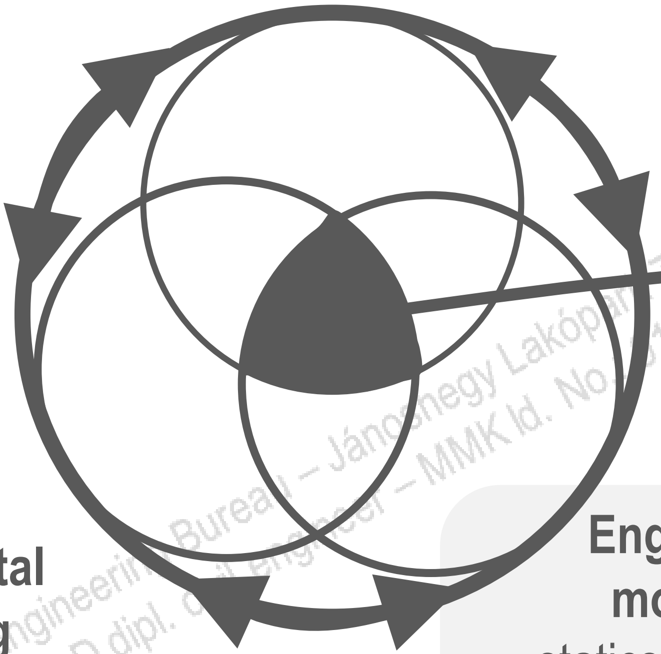
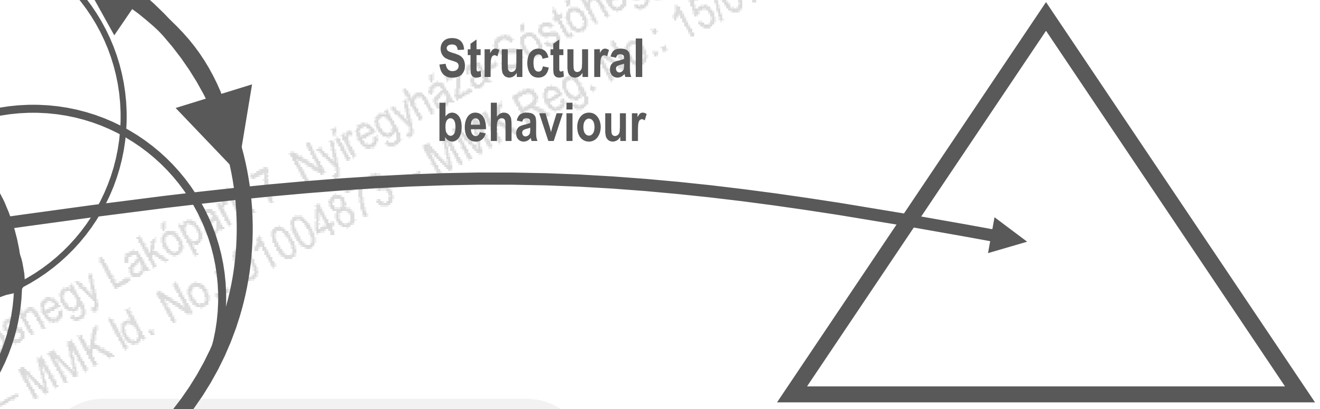
Modeling of structural behaviour of RC members

Numerical modelling
linear, non linear, plastic, 1st order theory, 2nd order theory, etc.

Material properties
homogen, inhomogen, izotropic, anizotropic
elastic linear, non linear elastic,
plastic, viscous, reological properties, etc.



Structural behaviour



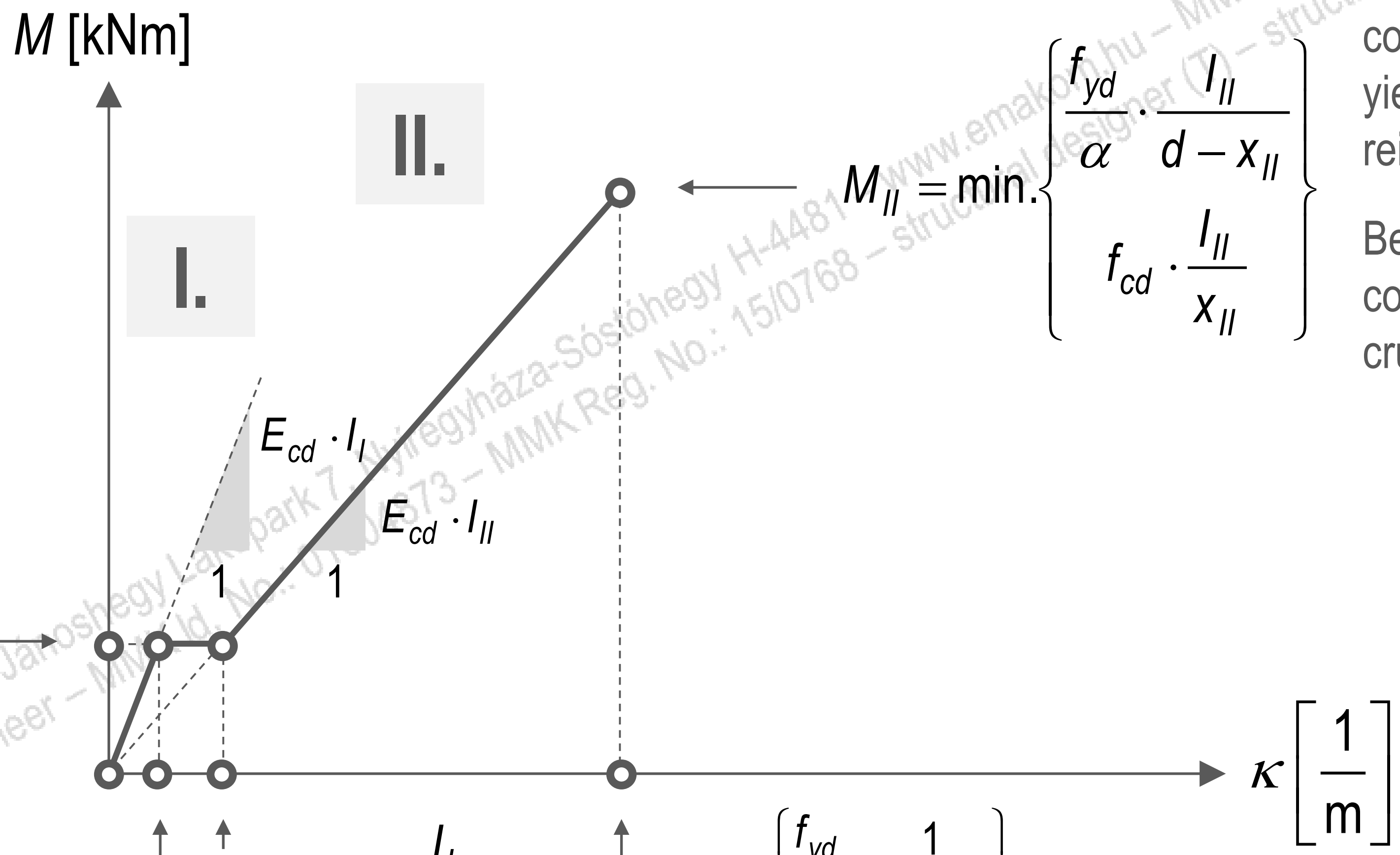
Experimental modelling
full-scale,
non full-scale, etc.

Engineering modelling
statics, strength of materials, elasticity, plasticity, dynamics, etc.

Size effect

Environment
effects, loads,
durability properties, etc.

RC cross-section in the uncracked and elastic/cracked state of stresses



$$M_{II} = \min.\begin{cases} \dfrac{f_{yd}}{\alpha} \cdot \dfrac{I_{II}}{d - x_{II}} \\ f_{cd} \cdot \dfrac{I_{II}}{x_{II}} \end{cases}$$

Bending moment corresponds to the yielding of reinforcement

Bending moment corresponds to the crushing of concrete

$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I}$$

$$M_I = M_{I-II} \rightarrow \kappa_I \cdot E_{cd} \cdot I_I = \kappa_{I-II} \cdot E_{cd} \cdot I_{II}$$

$$\kappa_{I-II} = \kappa_I \cdot \frac{I_I}{I_{II}}$$

$$\kappa_I = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h - x_I}$$

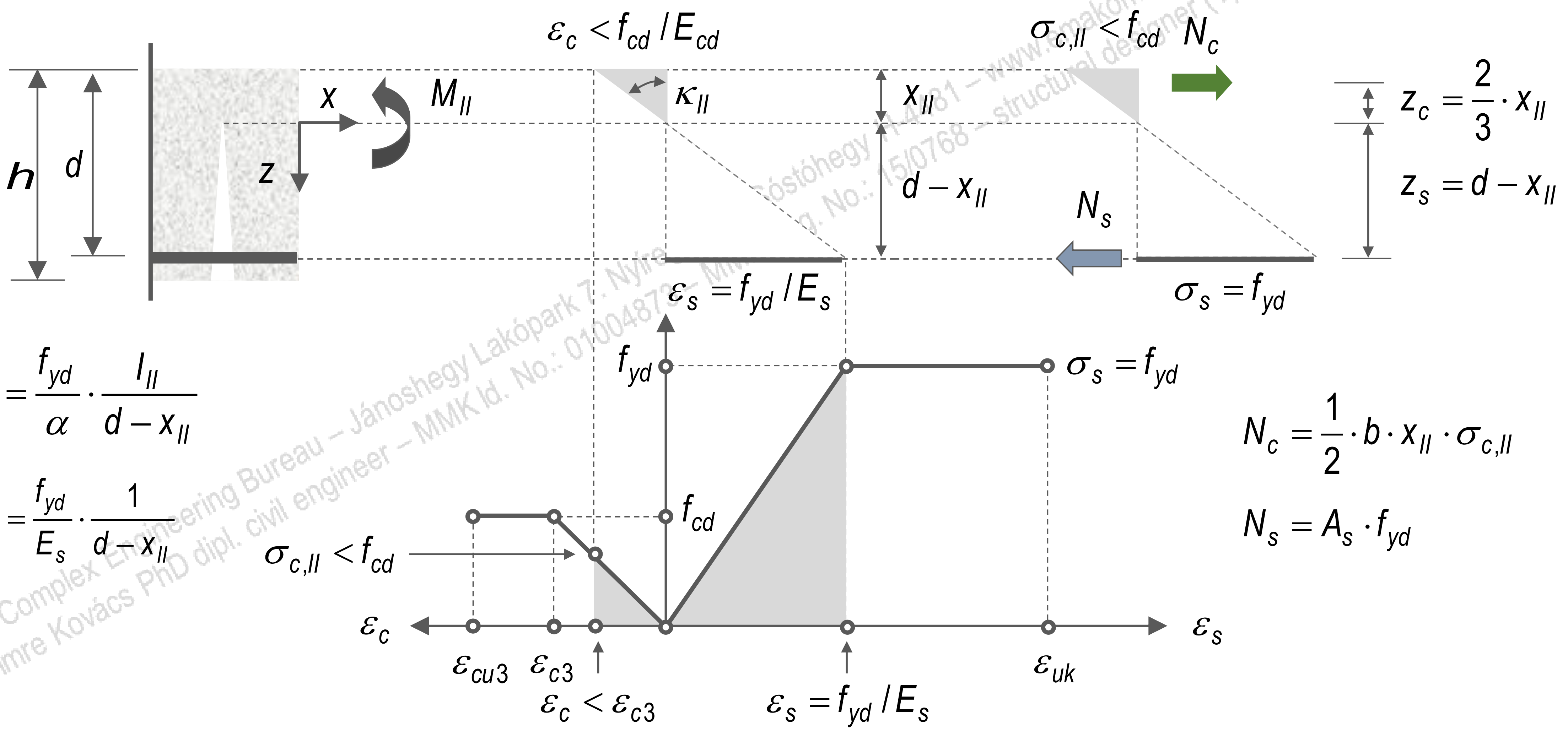
$$\kappa_{II} = \min.\begin{cases} \dfrac{f_{yd}}{E_s} \cdot \dfrac{1}{d - x_{II}} \\ \dfrac{f_{cd}}{E_{cd}} \cdot \dfrac{1}{x_{II}} \end{cases}$$

Curveture corresponds to the yielding of reinforcement

Curveture corresponds to the crushing of concrete

The first plastic phenomenon is the yielding of the reinforcemen – $\sigma_s = f_{yd}$

Geometry	Deformation, ε	Normal stress, σ



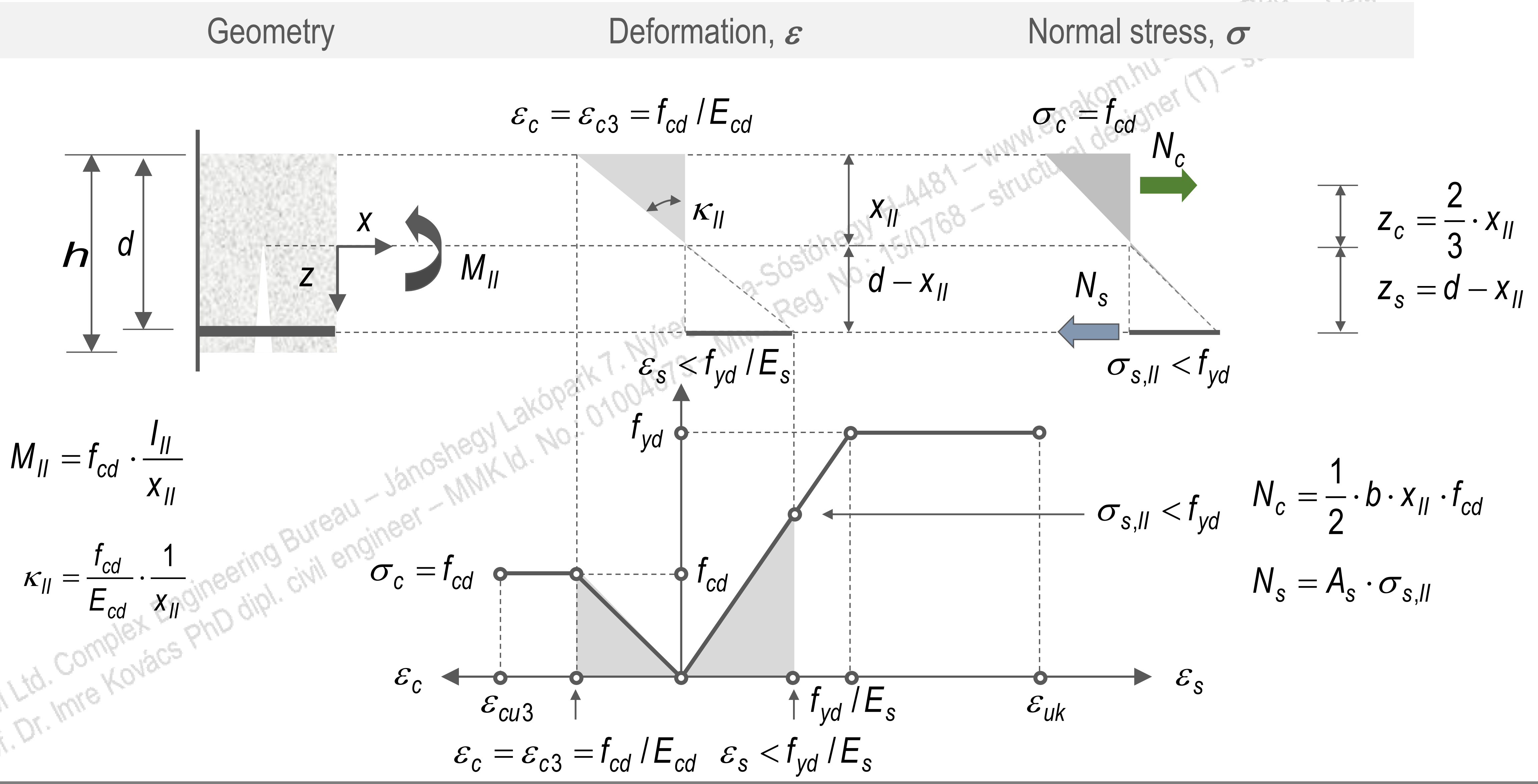
$$M_{II} = \frac{f_{yd}}{\alpha} \cdot \frac{I_{II}}{d - x_{II}}$$

$$\kappa_{II} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}}$$

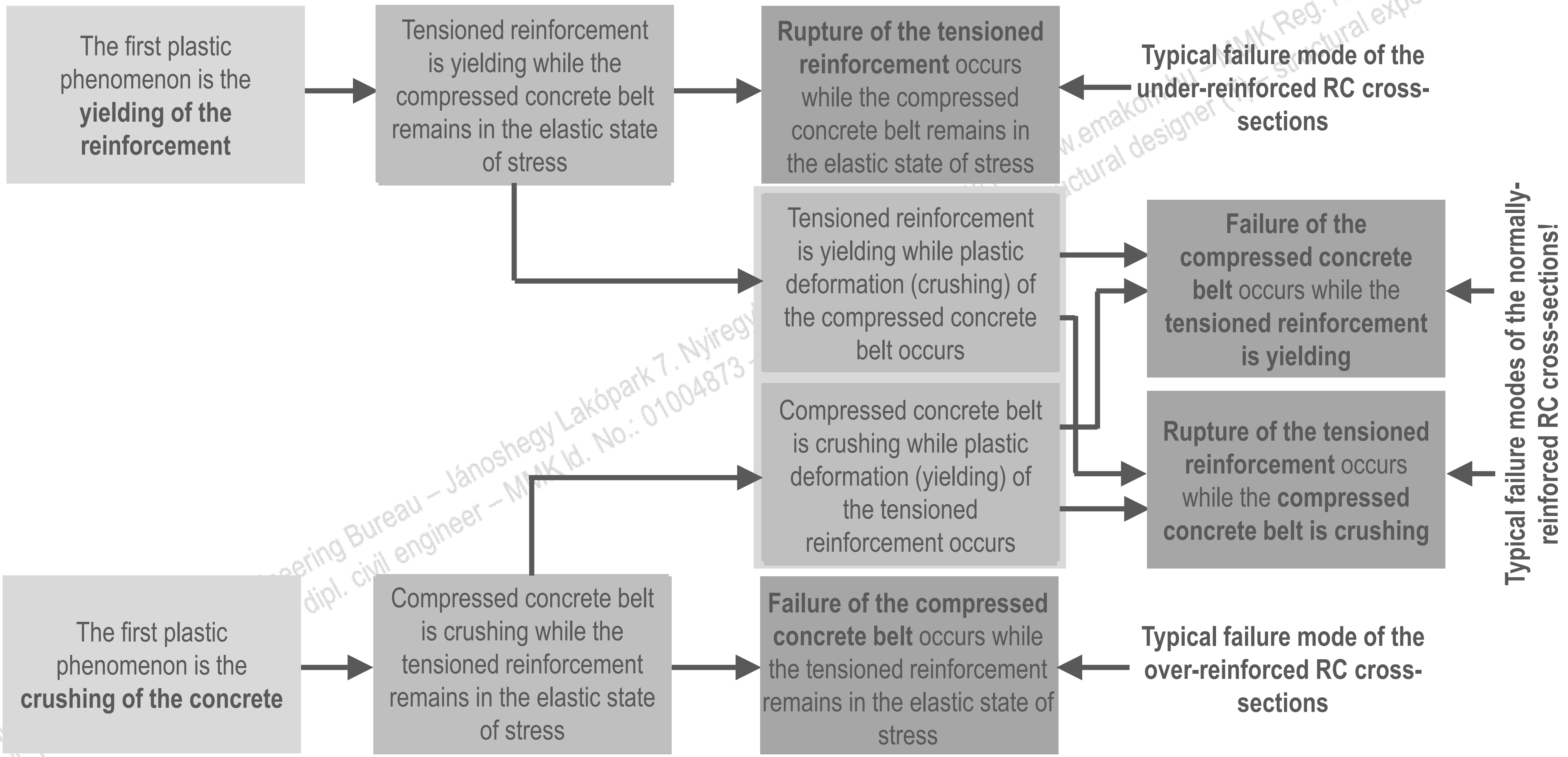
$$N_c = \frac{1}{2} \cdot b \cdot x_{II} \cdot \sigma_{c,II}$$

$$N_s = A_s \cdot f_{yd}$$

The first plastic phenomenon is the crushing of the concrete – $\sigma_c = f_{cd}$



Load process of reinforced concrete member in the intermediate state (bending)



Load process of reinforced concrete member in the intermediate state (bending)

$$\varepsilon_s = f_{yd}/E_s \quad \sigma_s = f_{yd} \quad \varepsilon_{c,II} <<< f_{cd}/E_{cd} \quad \sigma_{c,II} <<< f_{cd}$$

→

$$f_{yd}/E_s \le \varepsilon_s < \varepsilon_{uk} \quad \sigma_s = f_{yd} \quad \varepsilon_c << f_{cd}/E_{cd} \quad \sigma_c << f_{cd}$$

→

$$\varepsilon_s = \varepsilon_{uk}(\varepsilon_{ud}) \quad \sigma_s = f_{yd} \quad \varepsilon_c < f_{cd}/E_{cd} \quad \sigma_c < f_{cd}$$

← **Typical failure mode of the under-reinforced RC cross-sections**

$$f_{yd}/E_s \le \varepsilon_s < \varepsilon_{uk} \quad \sigma_s = f_{yd} \quad f_{cd}/E_{cd} \le \varepsilon_c < \varepsilon_{cu3} \quad \sigma_c = f_{cd}$$

$$f_{cd}/E_{cd} \le \varepsilon_c < \varepsilon_{cu3} \quad \sigma_c = f_{cd} \quad \varepsilon_s = f_{yd}/E_s \quad \sigma_s = f_{yd}$$

→

$$f_{yd}/E_s \le \varepsilon_s < \varepsilon_{uk} \quad \sigma_s = f_{yd} \quad \varepsilon_c = \varepsilon_{cu3} \quad \sigma_c = f_{cd}$$

$$f_{cd}/E_{cd} \le \varepsilon_c < \varepsilon_{cu3} \quad \sigma_c = f_{cd} \quad \varepsilon_s = \varepsilon_{uk}(\varepsilon_{ud}) \quad \sigma_s = f_{yd}$$

← **Typical failure modes of the normally-reinforced RC cross-sections!**

$$\varepsilon_c = f_{cd}/E_{cd} \quad \sigma_c = f_{cd} \quad \varepsilon_{s,II} <<< f_{yd}/E_s \quad \sigma_{s,II} <<< f_{yd}$$

→

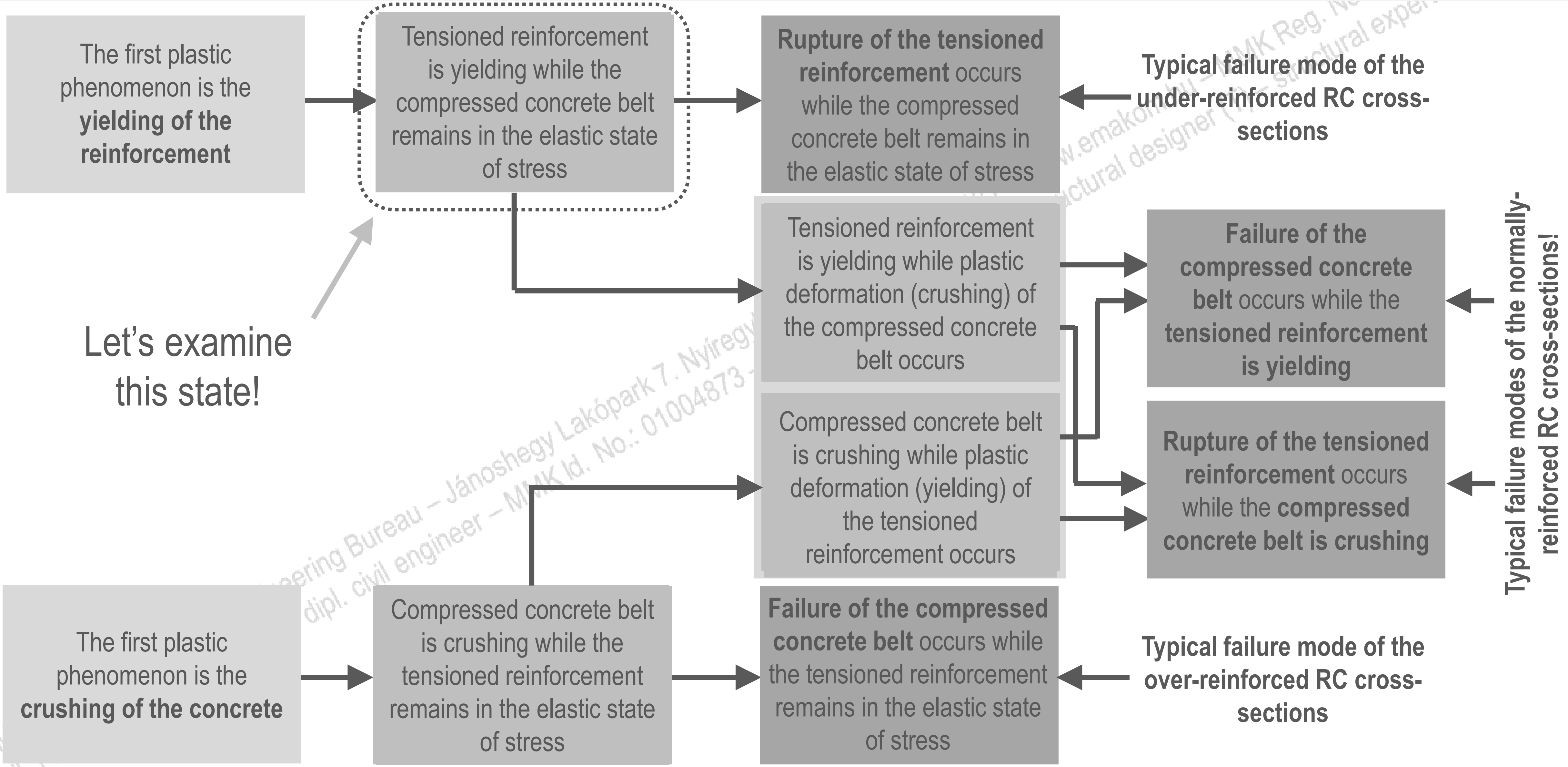
$$f_{cd}/E_{cd} \le \varepsilon_c < \varepsilon_{cu3} \quad \sigma_c = f_{cd} \quad \varepsilon_s << f_{yd}/E_s \quad \sigma_s << f_{yd}$$

→

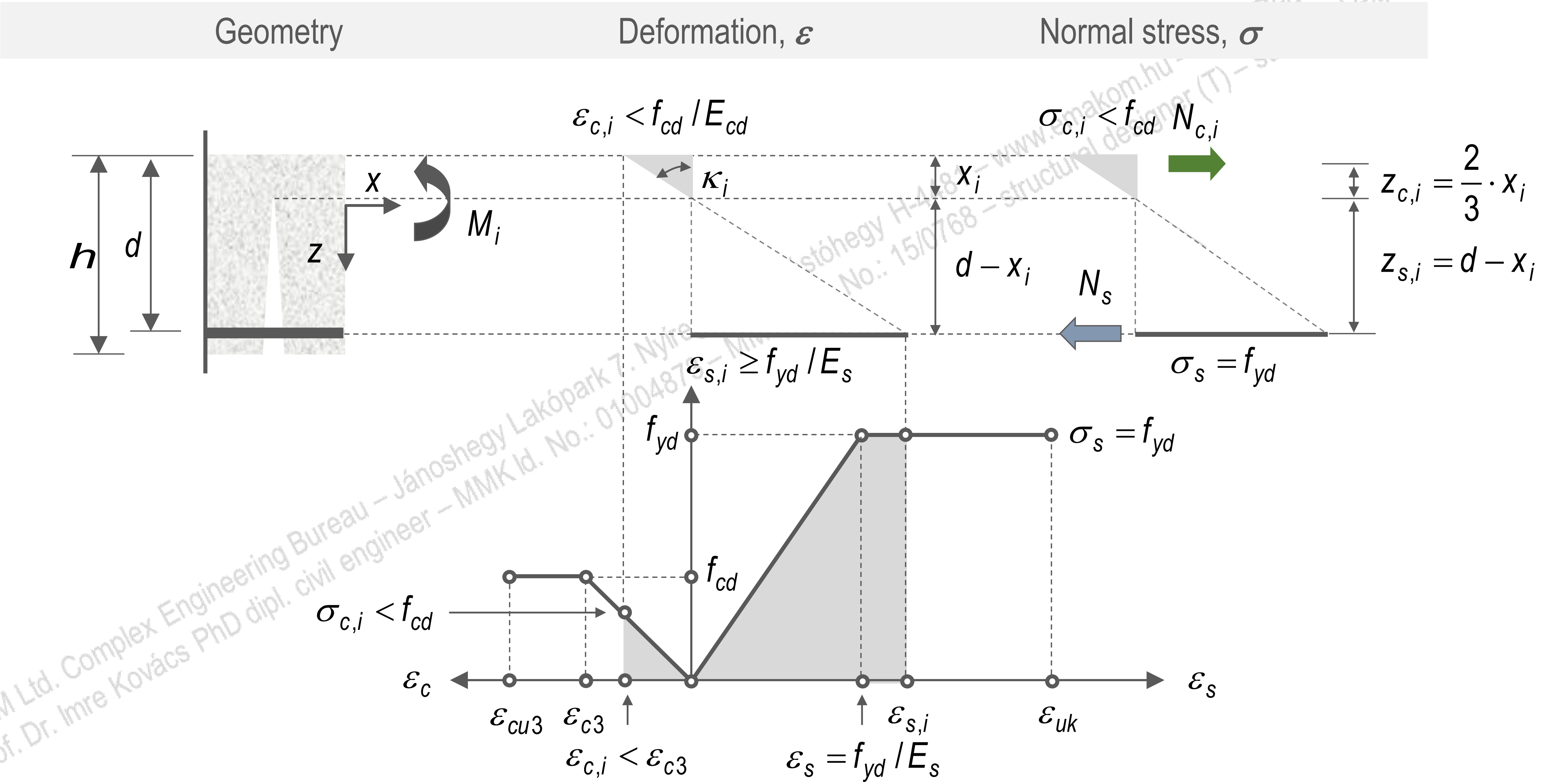
$$\varepsilon_c = \varepsilon_{cu3} \quad \sigma_c = f_{cd} \quad \varepsilon_s < f_{yd}/E_s \quad \sigma_s < f_{yd}$$

← **Typical failure mode of the over-reinforced RC cross-sections**

Reinforcement is yielding, concrete remains in the elastic state

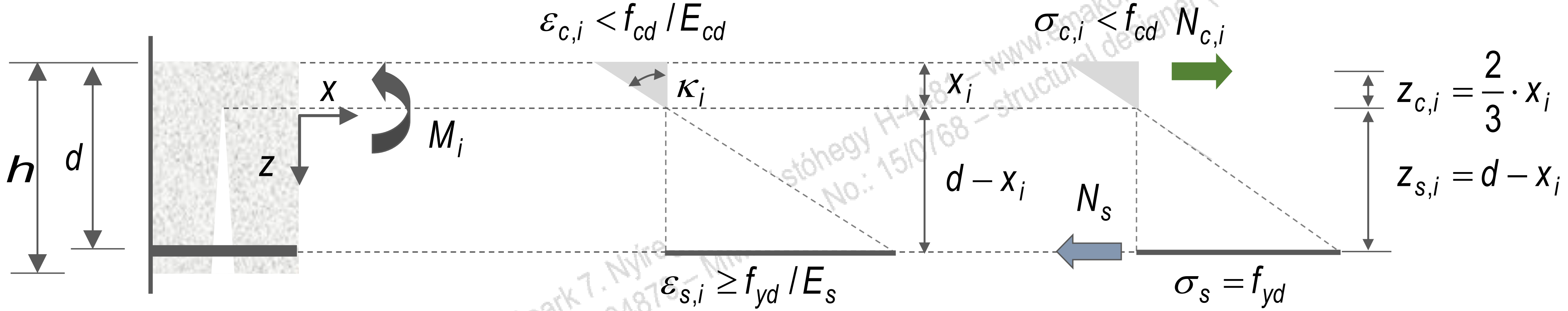


Reinforcement is yielding, concrete remains in the elastic state



Reinforcement is yielding, concrete remains in the elastic state

Geometry	Deformation, ε	Normal stress, σ



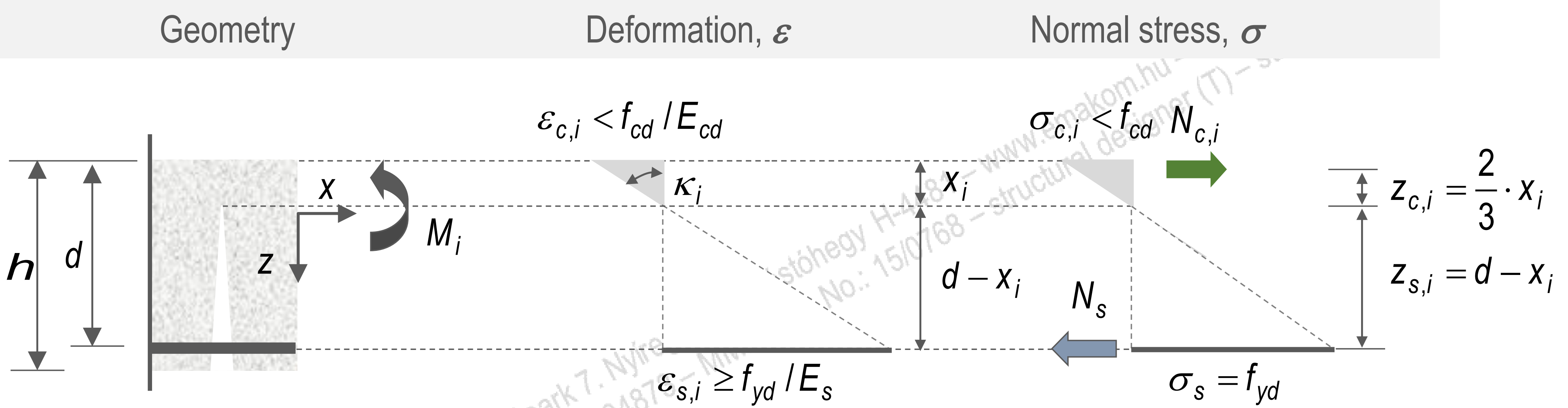
1. Horizontal force equilibrium: $\Sigma N = 0 \rightarrow N_{c,i} - N_s = 0$

- $N_{c,i} = \frac{1}{2} \cdot \sigma_{c,i} \cdot x_i \cdot b = \frac{1}{2} \cdot (\kappa_i \cdot x_i \cdot E_{cd}) \cdot x_i \cdot b = \frac{1}{2} \cdot \kappa_i \cdot x_i^2 \cdot E_{cd} \cdot b = \frac{1}{2} \cdot \varepsilon_{c,i} \cdot x_i \cdot E_{cd} \cdot b$
- $N_s = f_{yd} \cdot A_s$

$$\Sigma N = 0 \rightarrow \frac{1}{2} \cdot \varepsilon_{c,i} \cdot x_i \cdot E_{cd} \cdot b - f_{yd} \cdot A_s = 0 \rightarrow x_i = 2 \cdot \frac{f_{yd} \cdot A_s}{\varepsilon_{c,i} \cdot E_{cd} \cdot b}$$

First order equation for x_i, in which equation $\varepsilon_{c,i}$ is a parameter!!!
Solution can be made with taking different $\varepsilon_{c,i}$ values!!!

Reinforcement is yielding, concrete remains in the elastic state



2. Bending moment equilibrium: $\Sigma M = 0 \rightarrow M_i - N_{c,i} \cdot z_{c,i} - N_s \cdot z_{s,i} = 0$

- $M_i - \frac{1}{2} \cdot \varepsilon_{c,i} \cdot x_i \cdot E_{cd} \cdot b \cdot \frac{2}{3} \cdot x_i - f_{yd} \cdot A_s \cdot (d - x_i) = 0$
- $M_i = \frac{1}{2} \cdot \varepsilon_{c,i} \cdot x_i \cdot E_{cd} \cdot b \cdot \frac{2}{3} \cdot x_{c,i} + f_{yd} \cdot A_s \cdot (d - x_i)$

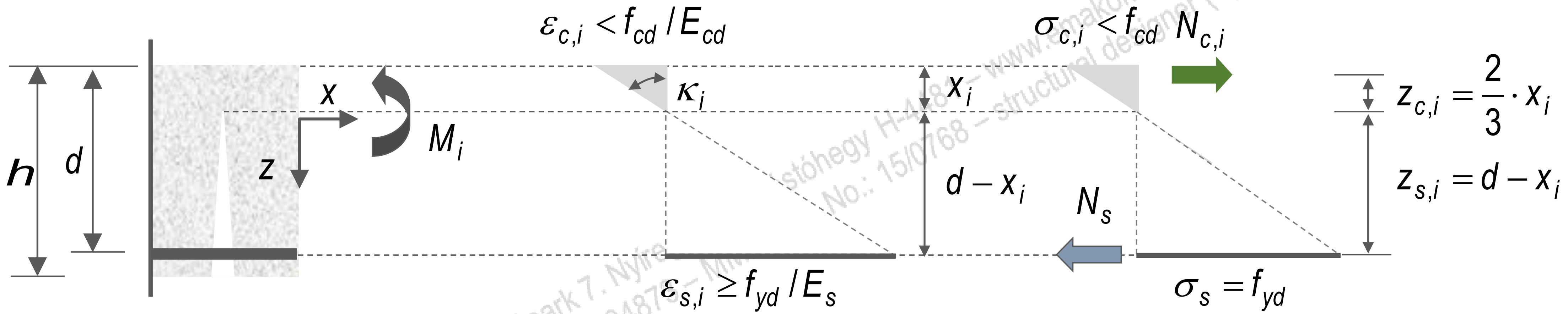
$$M_i = \frac{1}{3} \cdot E_{cd} \cdot b \cdot \varepsilon_{c,i} \cdot x_i^{2} - f_{yd} \cdot A_s \cdot x_i + f_{yd} \cdot A_s \cdot d$$

$$\kappa_i = \frac{\varepsilon_{c,i}}{x_i}$$

Taking x_i and $\varepsilon_{c,i}$ values the corresponding bending moment M_i, together with the curvature κ_i, can be determined!!!

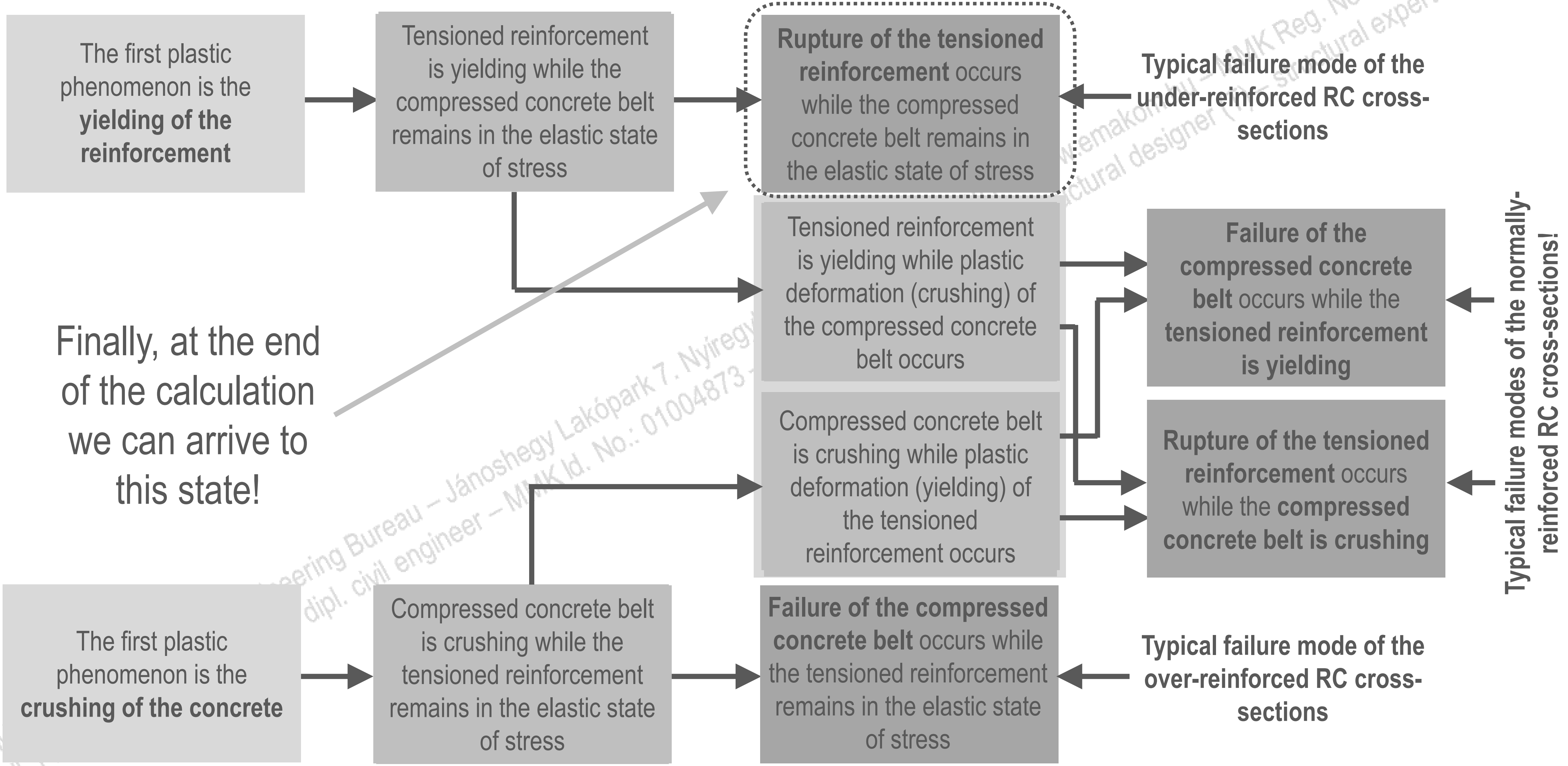
Reinforcement is yielding, concrete remains in the elastic state

Geometry | Deformation, ε | Normal stress, σ



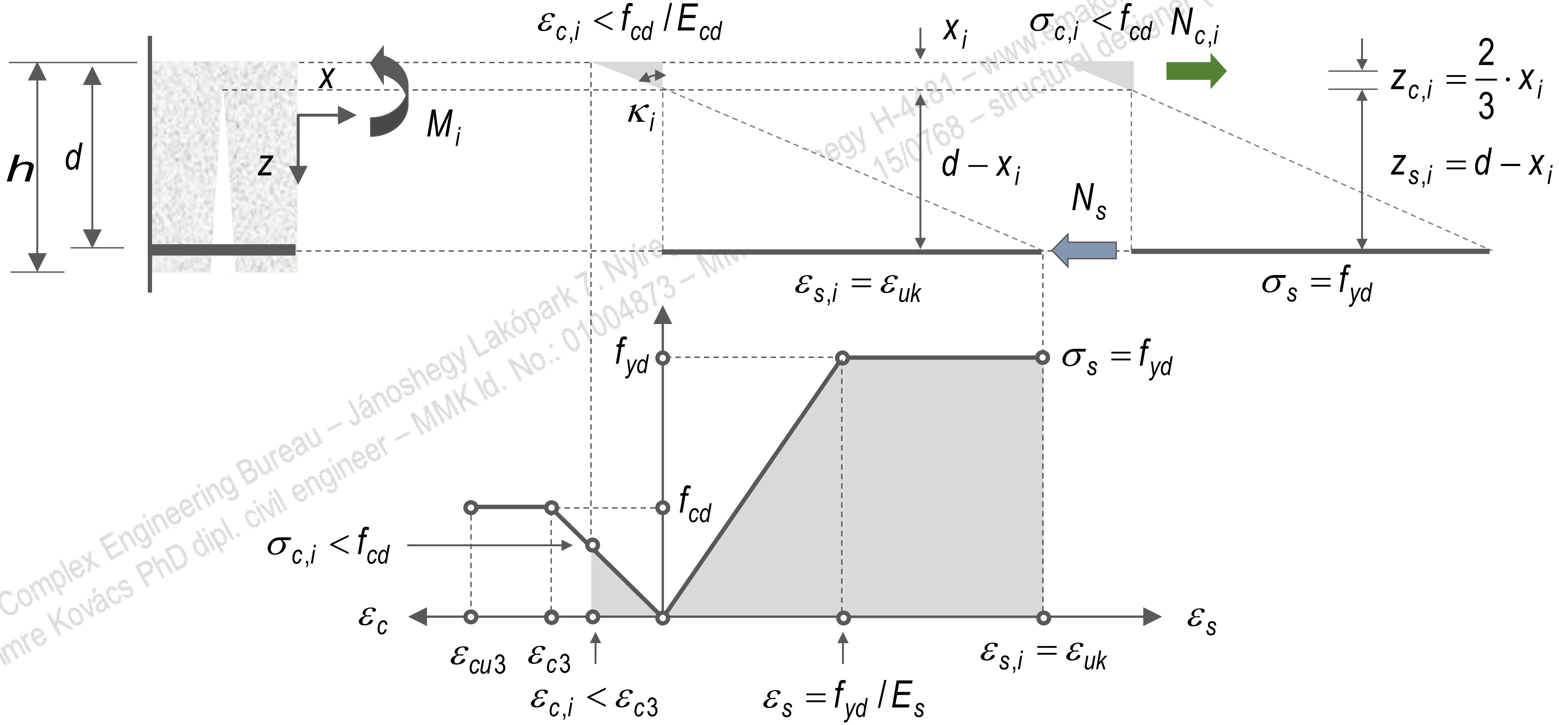
$\varepsilon_{c,i} [‰]$	$x_i [mm]$	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	$M_i [kNm]$	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i} [‰]$
$\varepsilon_{c,II}$				$\varepsilon_s = f_{yd} / E_s$
$\varepsilon_{c,i}$				Control for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c,i} < \varepsilon_{c3} = 1{,}75 ‰$				$\max. \varepsilon_{s,i} < \varepsilon_{uk} (\varepsilon_{ud})$!!!

Rupture of the reinforcement, concrete remains in the elastic state



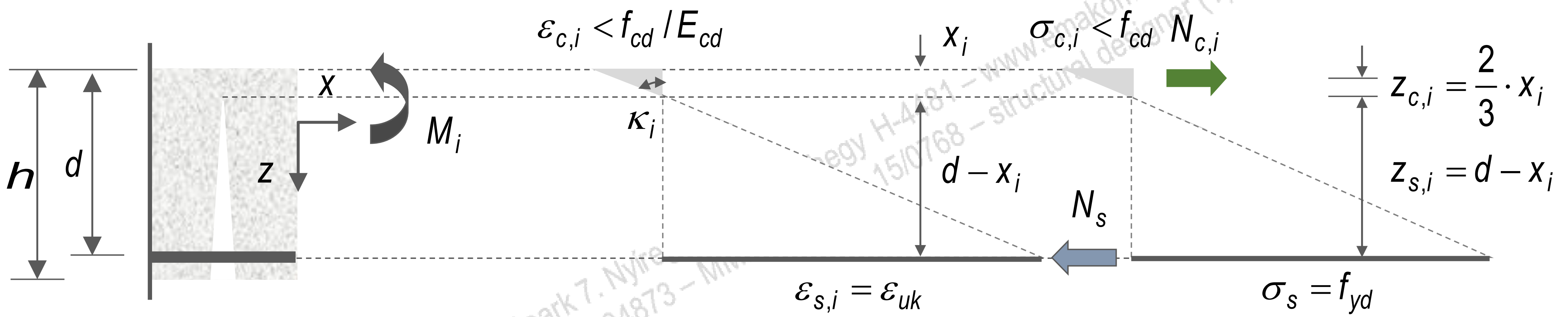
Rupture of the reinforcement, concrete remains in the elastic state

Geometry	Deformation, ε	Normal stress, σ



Rupture of the reinforcement, concrete remains in the elastic state

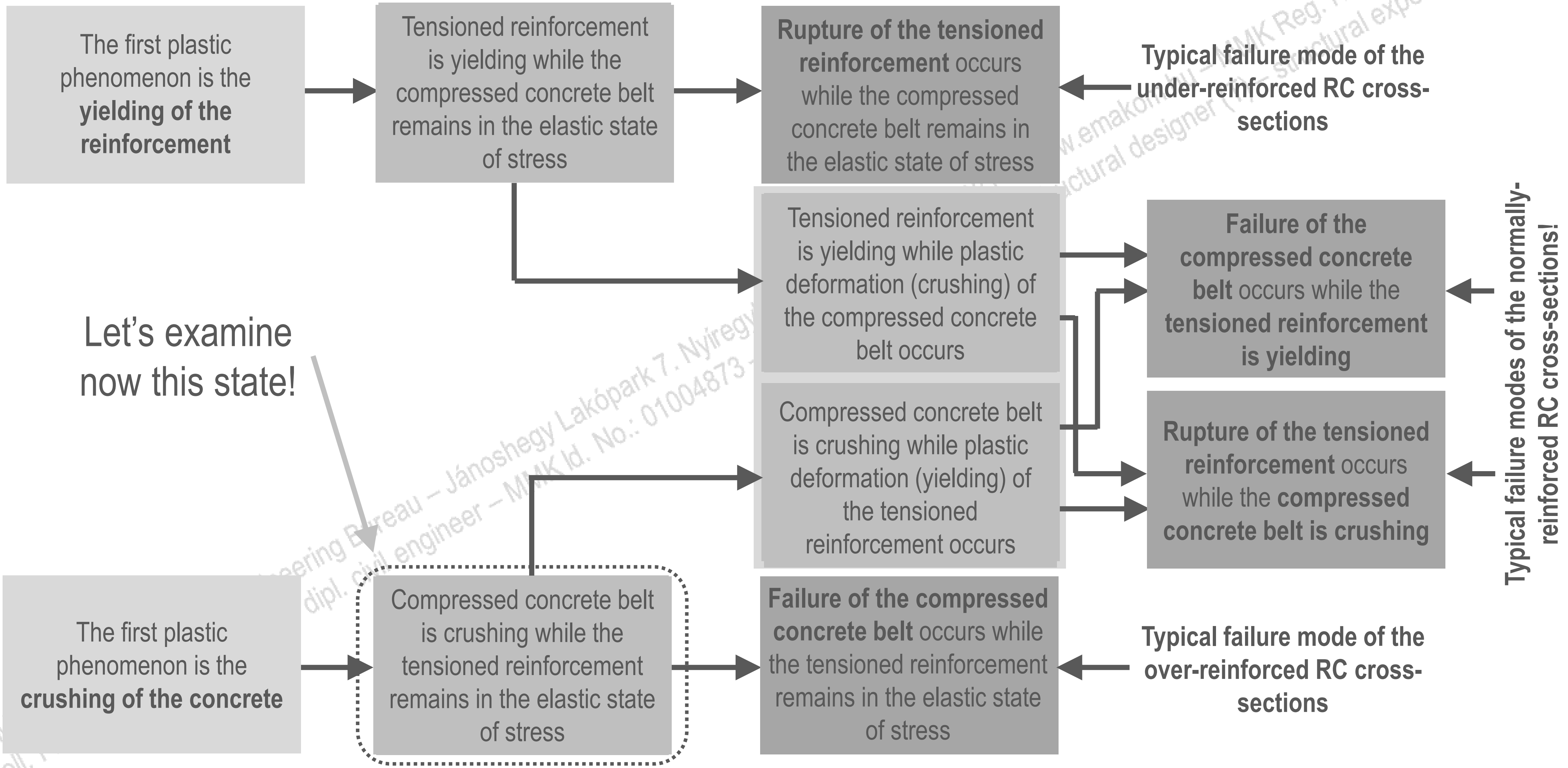
Geometry | Defromation, ε | Normal stress, σ



$\varepsilon_{c,i}$ [‰]	x_i [mm]	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	M_i [kNm]	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i}$ [‰]
$\varepsilon_{c,II}$				$\varepsilon_s = f_{yd} / E_s$
$\varepsilon_{c,i}$				Control for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c,i} < \varepsilon_{c3} = 1{,}75$ ‰				$\varepsilon_{s,i} = \varepsilon_{uk} (\varepsilon_{ud})$!!!

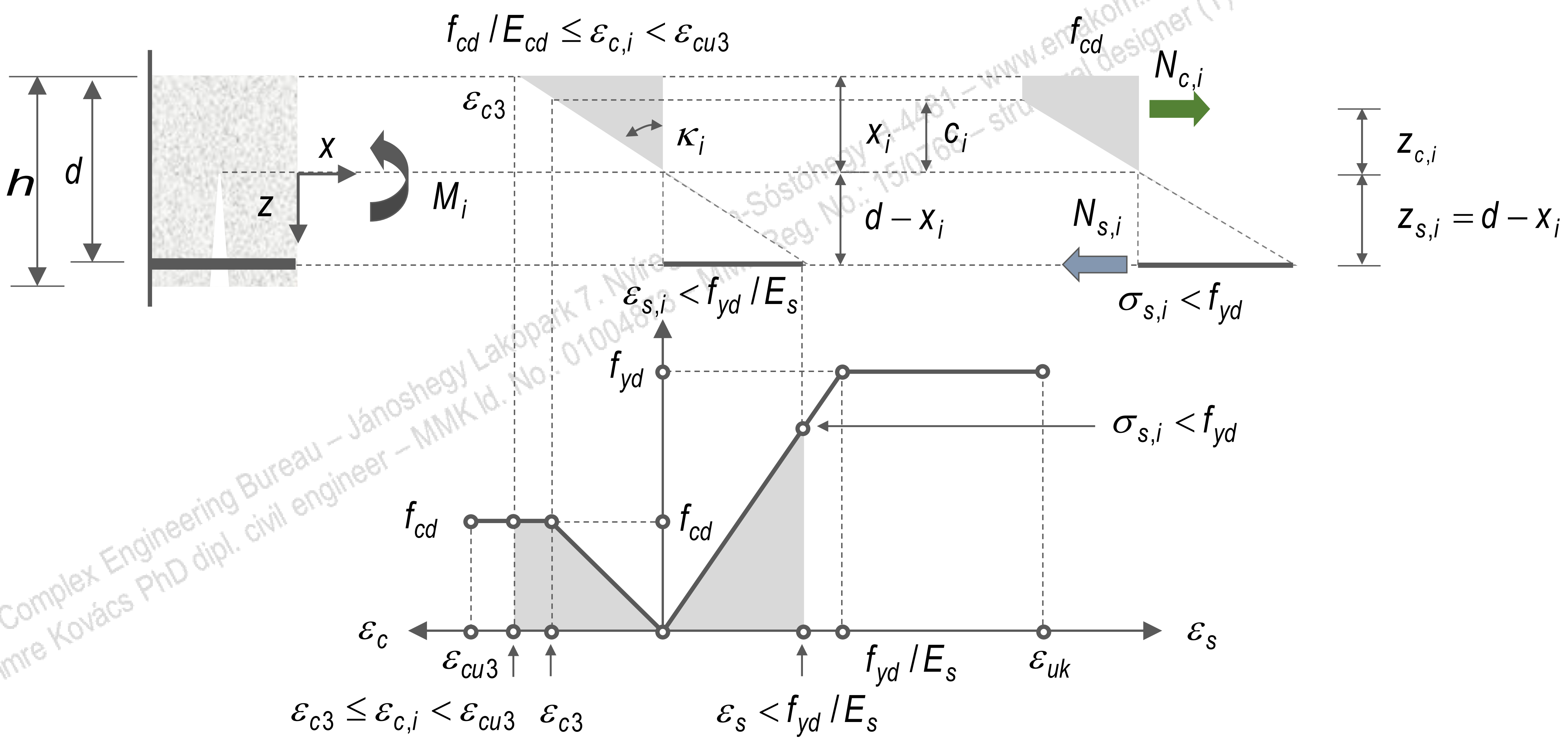
Typical failure mode of the under-reinforced RC cross-sections!

Concrete compressed belt is crushing, reinforcement remains in the elastic state



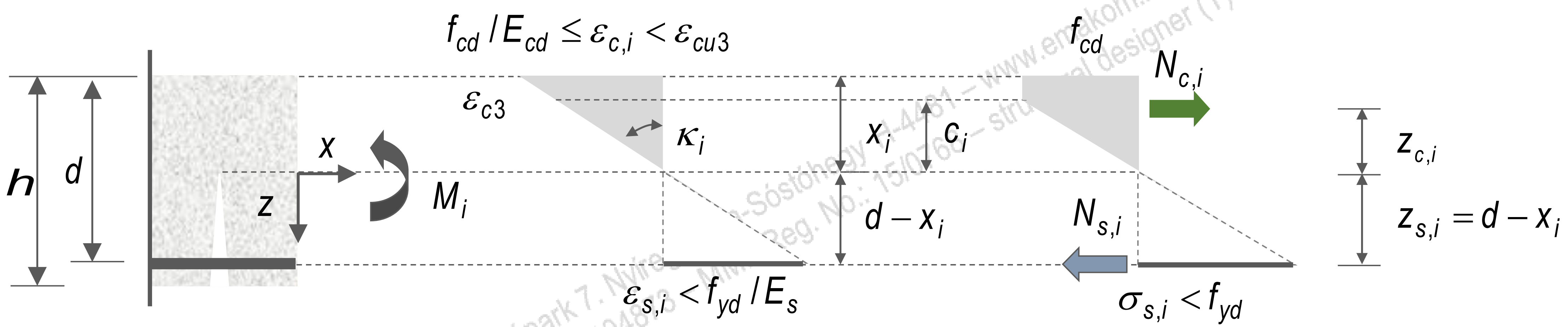
Concrete compressed belt is crushing, reinforcement remains in the elastic state

Geometry | Deformation, ε | Normal stress, σ



Concrete compressed belt is crushing, reinforcement remains in the elastic state

Geometry | Defromation, ε | Normal stress, σ



1. Horizontal force equilibrium: $\Sigma N = 0 \rightarrow N_{c,i} - N_s = 0$

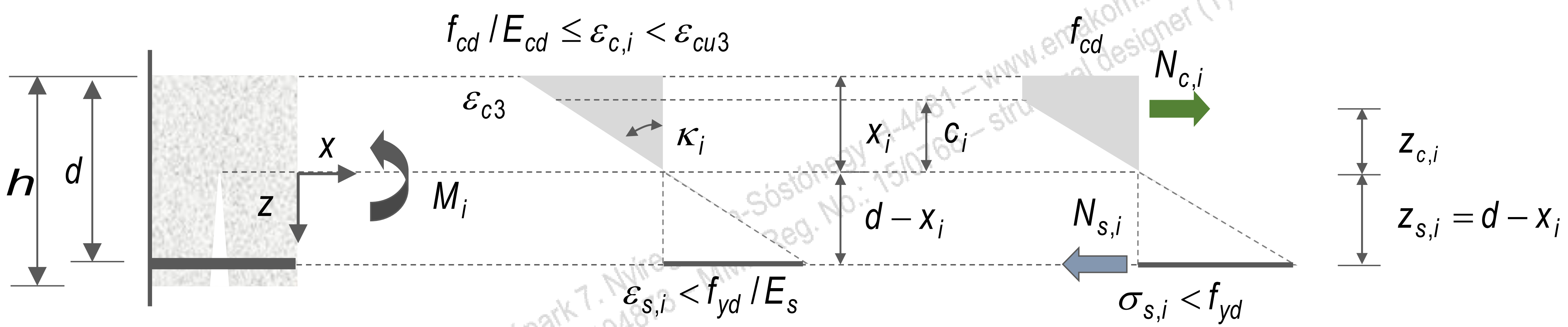
- $N_{c,i} = f_{cd} \cdot b \cdot x_i - \frac{1}{2} \cdot f_{cd} \cdot b \cdot c_i = f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{c_i}{x_i}\right) = f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right)$
- $N_{s,i} = \sigma_{s,i} \cdot A_s = \varepsilon_{s,i} \cdot E_s \cdot A_s = \varepsilon_{c,i} \cdot \frac{d - x_i}{x_i} \cdot E_s \cdot A_s = \varepsilon_{c,i} \cdot \frac{d}{x_i} \cdot E_s \cdot A_s - \varepsilon_{c,i} \cdot d \cdot E_s \cdot A_s$

Second order equation for x_i, in which equation $\varepsilon_{c,i}$ is a parameter!!!
Solution can be made with taking different $\varepsilon_{c,i}$ values!!!

$$\Sigma N = 0 \rightarrow \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right) \cdot x_i^2 + \varepsilon_{c,i} \cdot \frac{A_s \cdot E_s}{b \cdot f_{cd}} \cdot x_i - \varepsilon_{c,i} \cdot \frac{d \cdot A_s \cdot E_s}{b \cdot f_{cd}} = 0 \rightarrow x_i$$

Concrete compressed belt is crushing, reinforcement remains in the elastic state

Geometry	Deformation, ε	Normal stress, σ



2. Bending moment equilibrium: $\Sigma M = 0 \quad \rightarrow \quad M_i - N_{c,i} \cdot z_{c,i} - N_{s,i} \cdot z_{s,i} = 0$

- $$N_{c,i} \cdot z_{c,i} = f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right) \cdot \frac{2}{3} \cdot x_i = f_{cd} \cdot b \cdot \left(\frac{2 \cdot \varepsilon_{c,i} - \varepsilon_{c3}}{3 \cdot \varepsilon_{c,i}}\right) \cdot x_i^2$$
- $$N_{s,i} \cdot z_{s,i} = \varepsilon_{c,i} \cdot \frac{d - x_i}{x_i} \cdot E_s \cdot A_s \cdot (d - x_i) = \varepsilon_{c,i} \cdot \frac{(d - x_i)^2}{x_i} \cdot E_s \cdot A_s$$

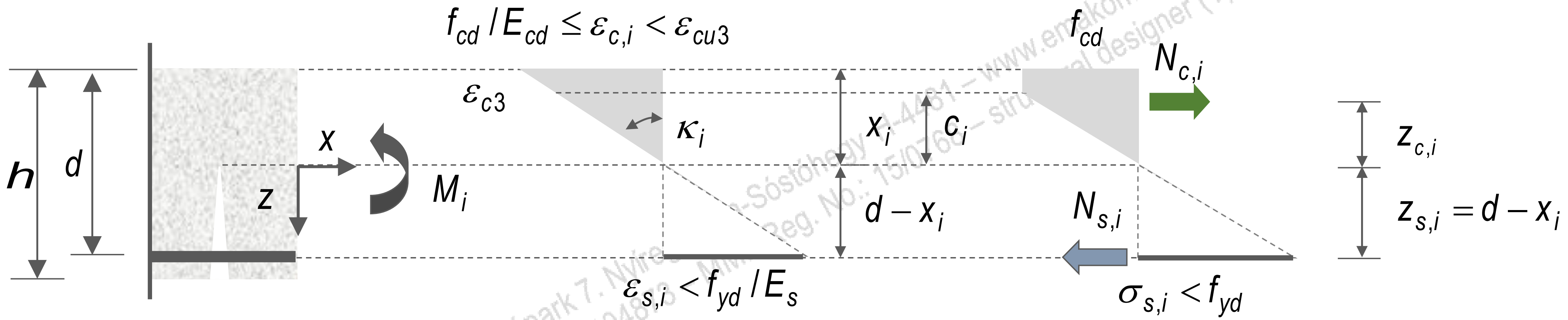
Taking x_i and $\varepsilon_{c,i}$ values the corresponding bending moment M_i, together with the curvature κ_i, can be determined!!!

$$M_i = f_{cd} \cdot b \cdot \left(\frac{2 \cdot \varepsilon_{c,i} - \varepsilon_{c3}}{3 \cdot \varepsilon_{c,i}}\right) \cdot x_i^2 + \varepsilon_{c,i} \cdot \frac{(d - x_i)^2}{x_i} \cdot E_s \cdot A_s$$

$$\kappa_i = \frac{\varepsilon_{c,i}}{x_i}$$

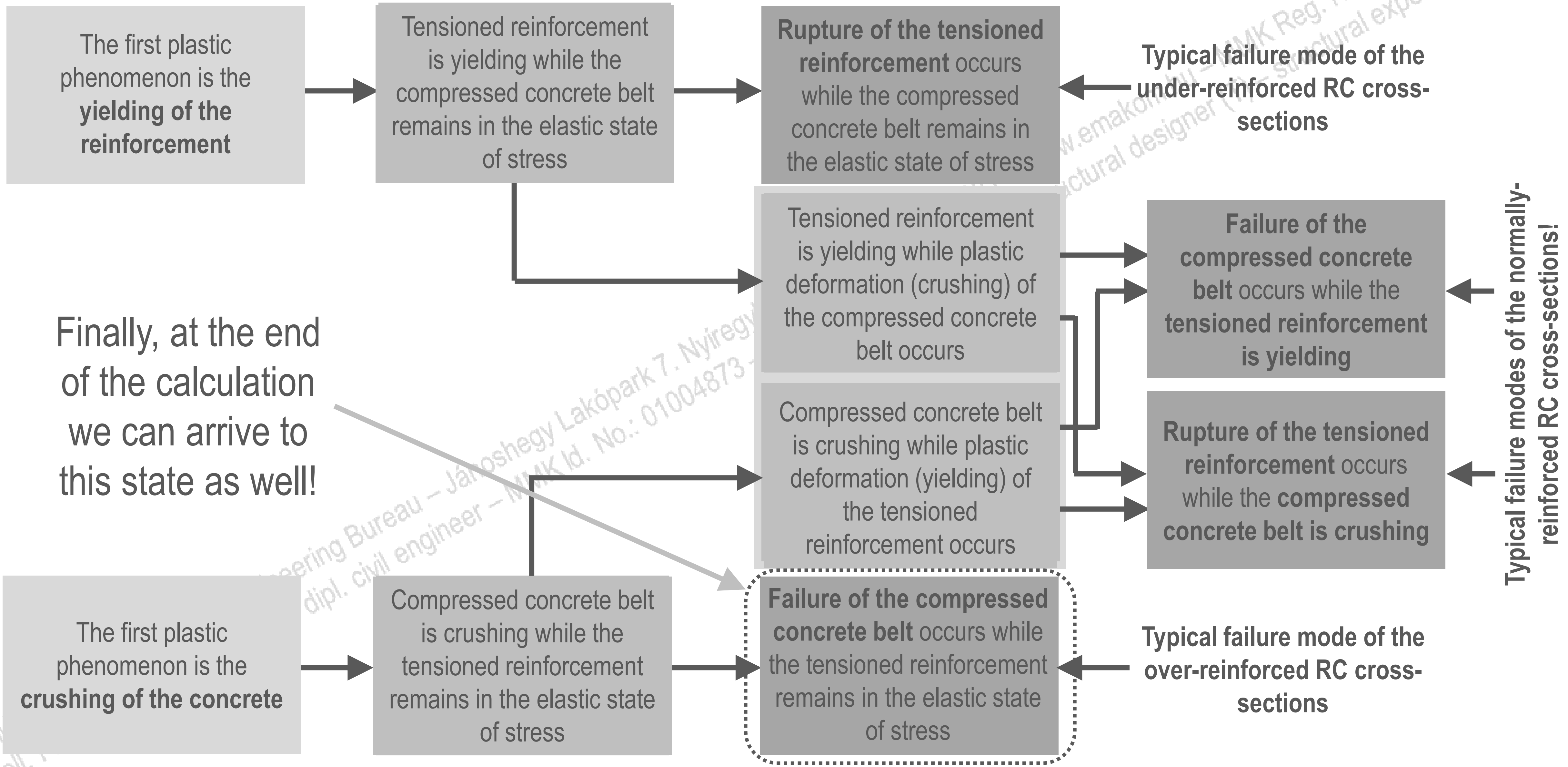
Concrete compressed belt is crushing, reinforcement remains in the elastic state

Geometry | Deformation, ε | Normal stress, σ



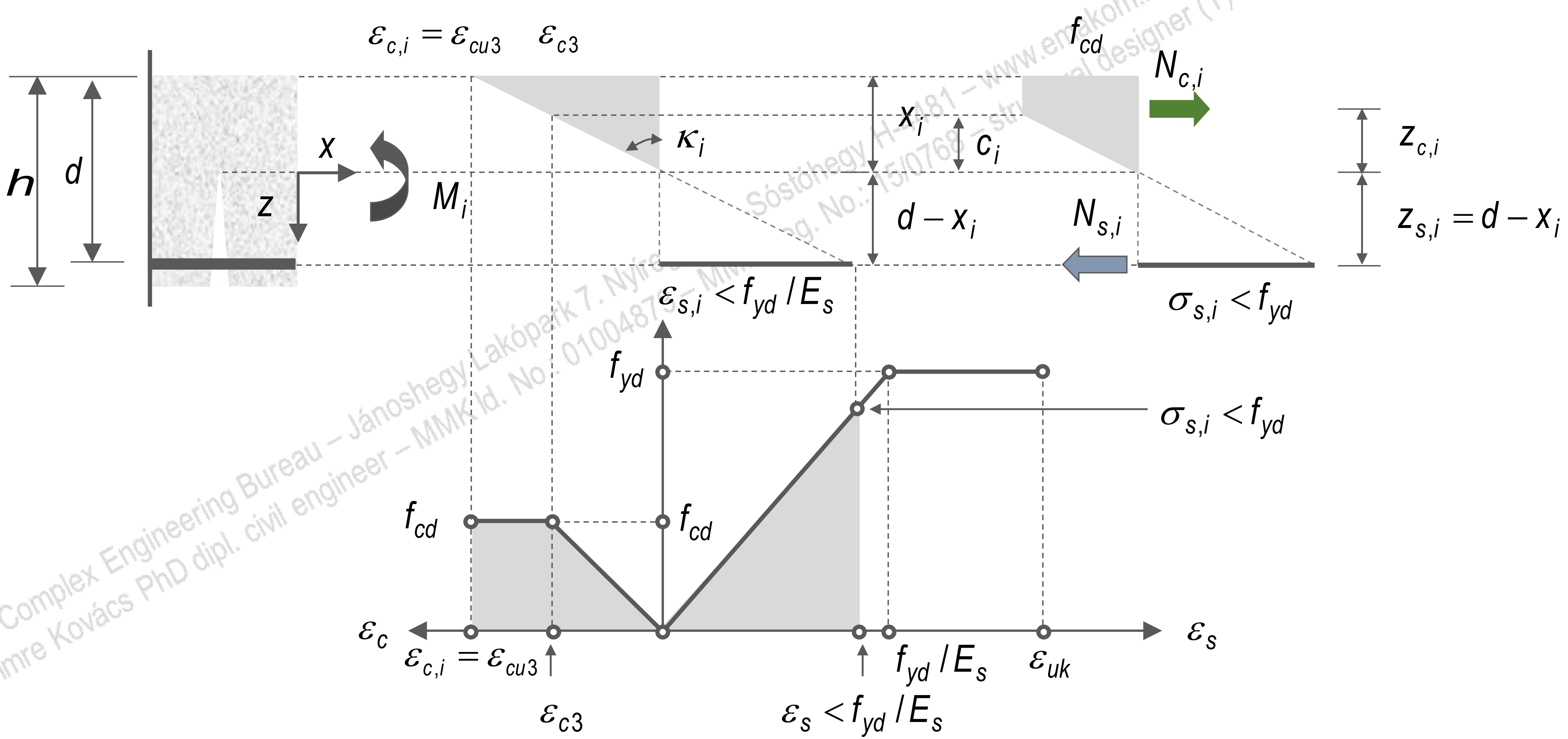
$\varepsilon_{c,i} [‰]$	x_i [mm]	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	M_i [kNm]	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i} [‰]$
$\varepsilon_{c,i} \ge \varepsilon_{c3}$				$\varepsilon_{s,II}$
$\varepsilon_{c,i}$				Control for $\varepsilon_{s,i}$ value!
….				…
max. $\varepsilon_{c,i} < \varepsilon_{cu3}$!!!				$\varepsilon_s < f_{yd} / E_s$

Failure of the compressed concrete belt, tensioned reinforcement remains in the elastic state



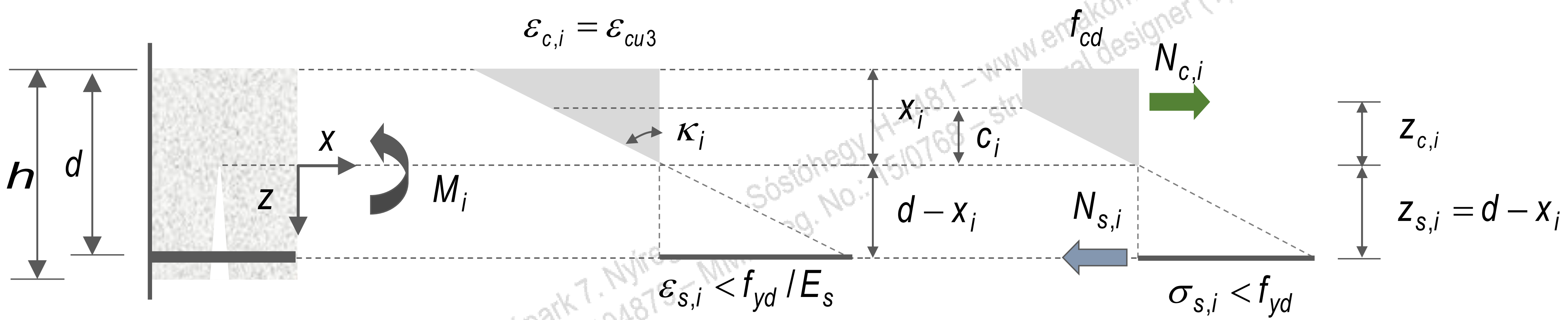
Failure of the compressed concrete belt, tensioned reinforcement remains in the elastic state

Geometry	Deformation, ε	Normal stress, σ



Failure of the compressed concrete belt, tensioned reinforcement remains in the elastic state

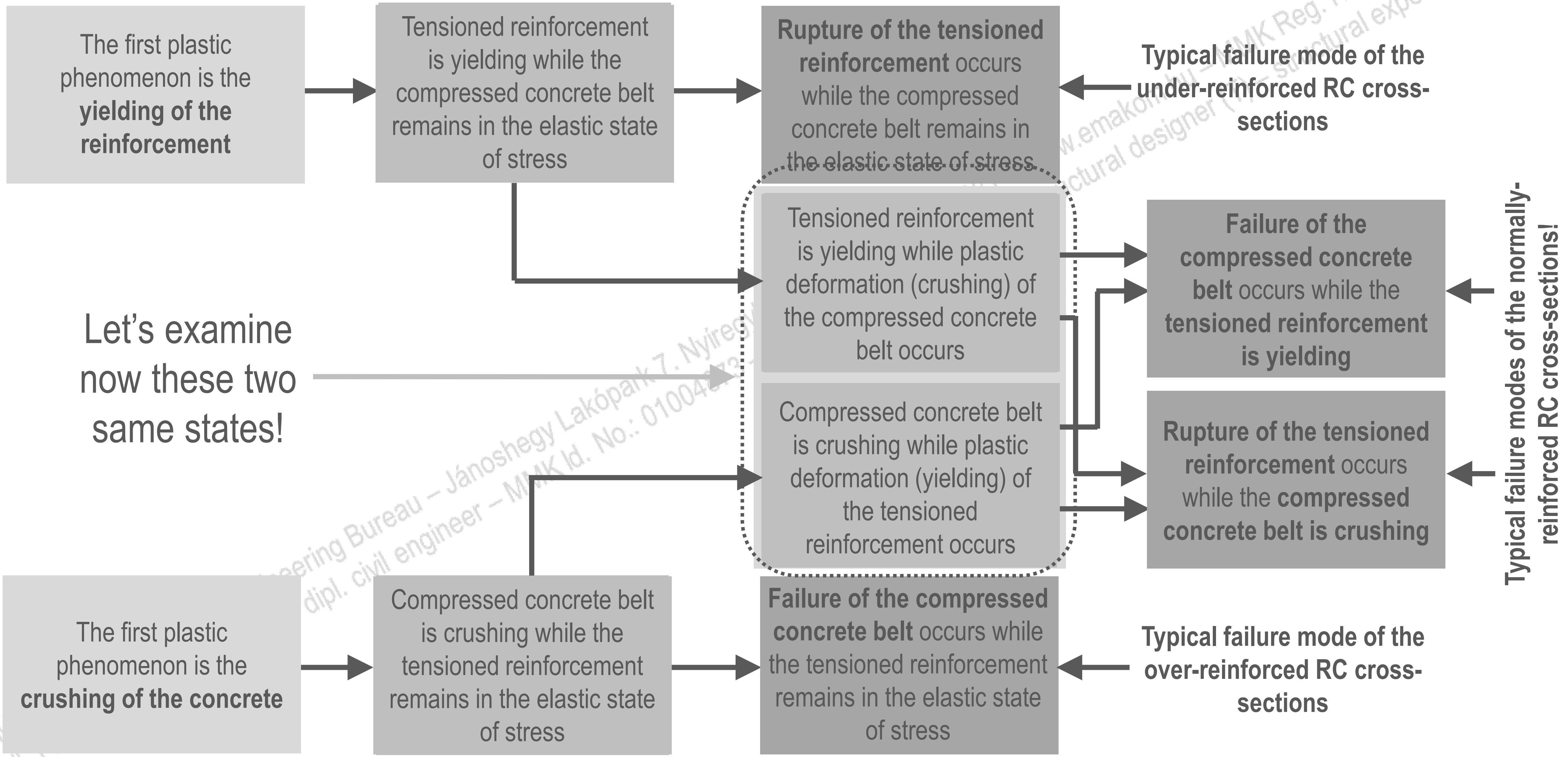
Geometry	Deformation, ε	Normal stress, σ



$\varepsilon_{c,i} [‰]$	$x_i [\text{mm}]$	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{\text{mm}}\right]$	$M_i [\text{kNm}]$	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i} [‰]$
$\varepsilon_{c,i} \geq \varepsilon_{c3}$				$\varepsilon_{s,II}$
$\varepsilon_{c,i}$				Control for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c,i} = \varepsilon_{cu3}$!!!				$\varepsilon_s < f_{yd} / E_s$

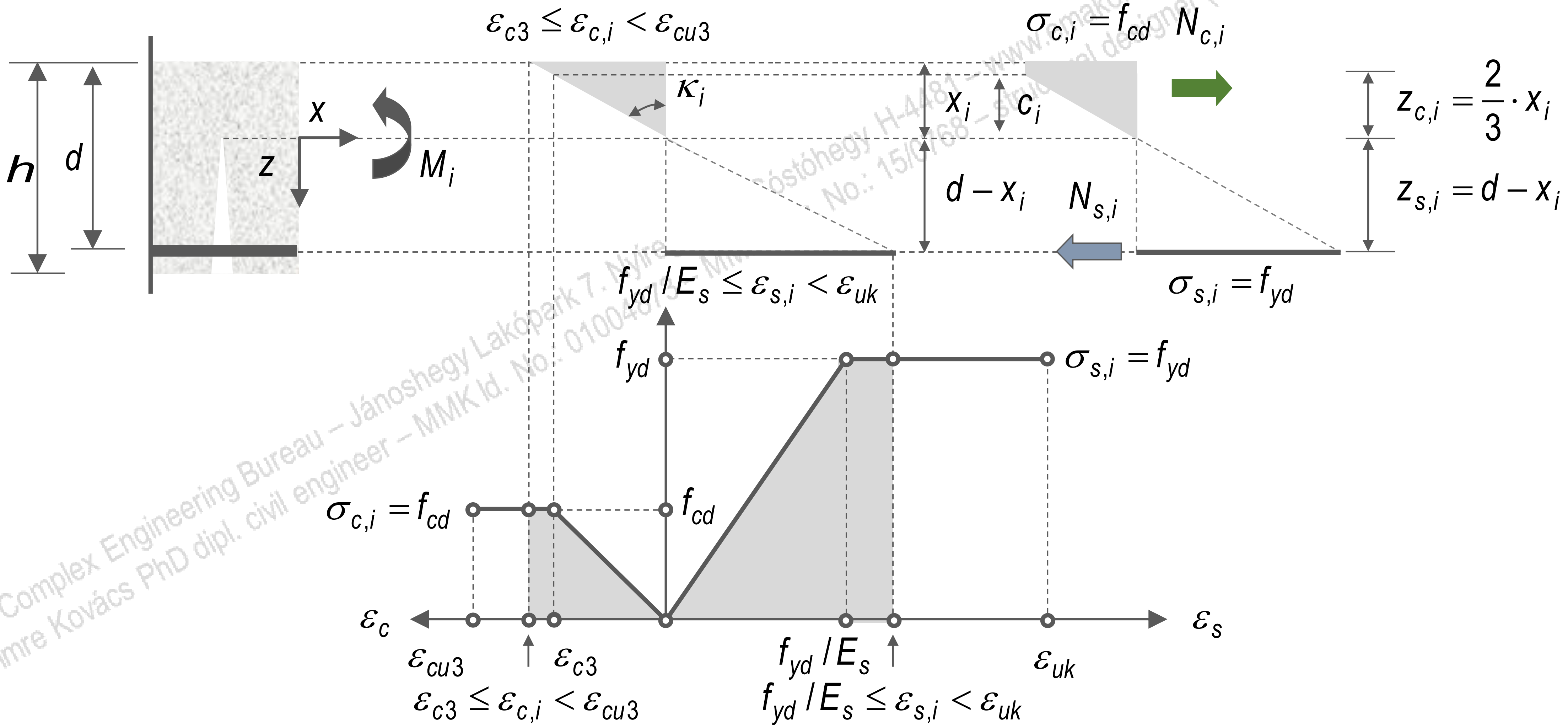
Typical failure mode of the over-reinforced RC cross-sections!

Reinforcement is yielding, compressed concrete belt is crushing



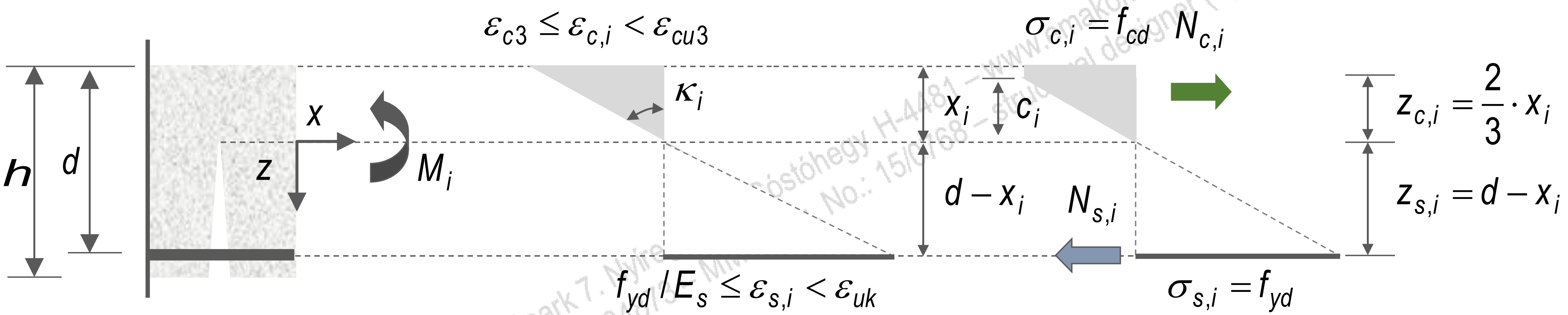
Reinforcement is yielding, compressed concrete belt is crushing

Geometry	Deformation, ε	Normal stress, σ



Reinforcement is yielding, compressed concrete belt is crushing

Geometry	Defromation, ε	Normal stress, σ



1. Horizontal force equilibrium: $\Sigma N = 0 \rightarrow N_{c,i} - N_{s,i} = 0$

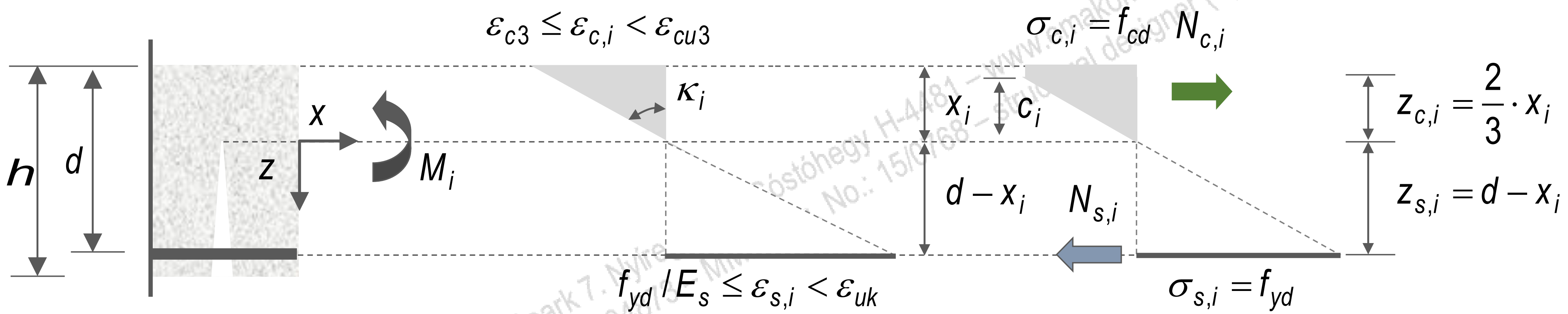
- $N_{c,i} = f_{cd} \cdot b \cdot x_i - \frac{1}{2} \cdot f_{cd} \cdot b \cdot c_i = f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{c_i}{x_i}\right) = f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right)$
- $N_{s,i} = f_{yd} \cdot A_s$

First order equation for x_i, in which equation $\varepsilon_{c,i}$ is a parameter!!!
Solution can be made with taking different $\varepsilon_{c,i}$ values!!!

$$\Sigma N = 0 \rightarrow f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right) - f_{yd} \cdot A_s = 0 \rightarrow x_i = \frac{f_{cd} \cdot b}{f_{yd} \cdot A_s} \cdot \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right)$$

Reinforcement is yielding, compressed concrete belt is crushing

Geometry | Deformation, ε | Normal stress, σ



2. Bending moment equilibrium: $\Sigma M = 0 \rightarrow M_i - N_{c,i} \cdot z_{c,i} - N_{s,i} \cdot z_{s,i} = 0$

- $N_{c,i} \cdot z_{c,i} = f_{cd} \cdot b \cdot x_i \cdot \left(1 - \frac{1}{2} \cdot \frac{\varepsilon_{c3}}{\varepsilon_{c,i}}\right) \cdot \frac{2}{3} \cdot x_i = f_{cd} \cdot b \cdot \left(\frac{2 \cdot \varepsilon_{c,i} - \varepsilon_{c3}}{3 \cdot \varepsilon_{c,i}}\right) \cdot x_i^2$
- $N_{s,i} \cdot z_{s,i} = f_{yd} \cdot A_s \cdot (d - x_i)$

Taking $\boldsymbol{x_i}$ and $\boldsymbol{\varepsilon_{c,i}}$ values the corresponding bending moment $\boldsymbol{M_i}$, together with the curvature $\boldsymbol{\kappa_i}$, can be determined!!!

$$M_i = f_{cd} \cdot b \cdot \left(\frac{2 \cdot \varepsilon_{c,i} - \varepsilon_{c3}}{3 \cdot \varepsilon_{c,i}}\right) \cdot x_i^2 + f_{yd} \cdot A_s \cdot (d - x_i)$$

$$\kappa_i = \frac{\varepsilon_{c,i}}{x_i}$$

Reinforcement is already yielding, crushing of compressed concrete belt occurs

The first plastic phenomenon is the **yielding of the reinforcement**

Tensioned reinforcement is yielding while the compressed concrete belt remains in the elastic state of stress

Rupture of the tensioned reinforcement occurs while the compressed concrete belt remains in the elastic state of stress

Typical failure mode of the under-reinforced RC cross-sections

…the previously explained process can start with the crushing of the compressed concrete belt, at which point the tensioned reinforcement is already yealding…

Tensioned reinforcement is yielding while plastic deformation (crushing) of the compressed concrete belt occurs

Compressed concrete belt is crushing while plastic deformation (yielding) of the tensioned reinforcement occurs

Failure of the compressed concrete belt occurs while the **tensioned reinforcement is yielding**

Rupture of the tensioned reinforcement occurs while the **compressed concrete belt is crushing**

Typical failure modes of the normally-reinforced RC cross-sections!

The first plastic phenomenon is the **crushing of the concrete**

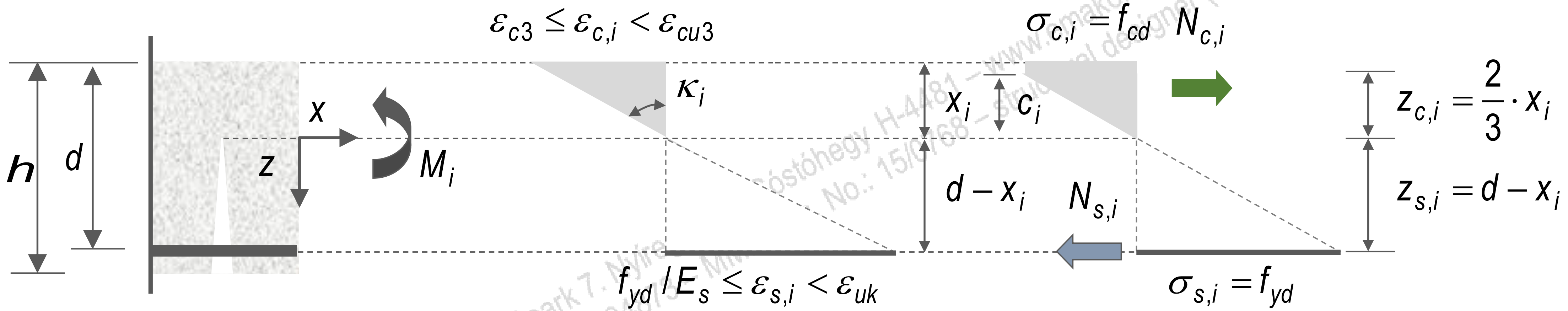
Compressed concrete belt is crushing while the tensioned reinforcement remains in the elastic state of stress

Failure of the compressed concrete belt occurs while the tensioned reinforcement remains in the elastic state of stress

Typical failure mode of the over-reinforced RC cross-sections

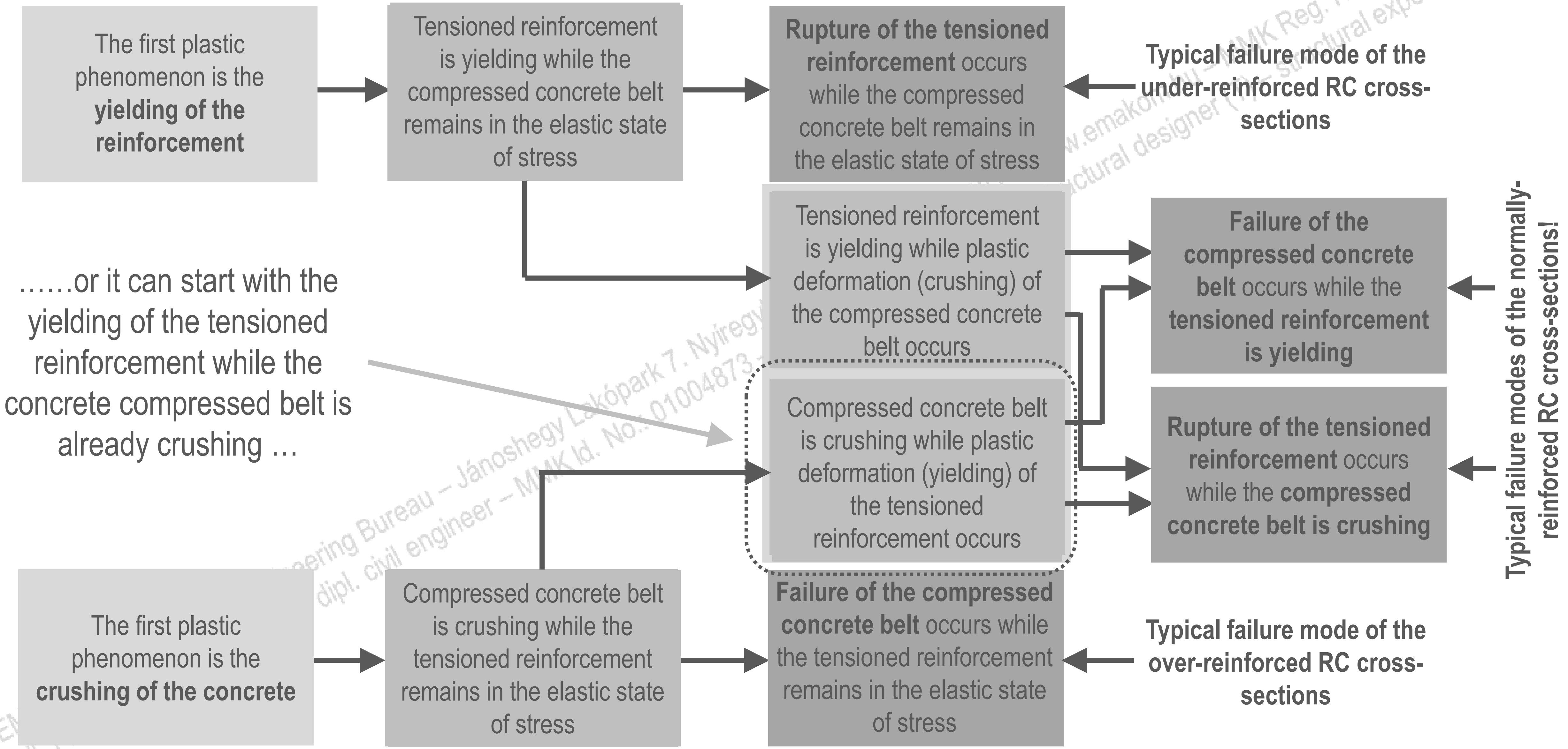
Reinforcement is already yielding, crushing of compressed concrete belt occurs

Geometry	Deformation, ε	Normal stress, σ



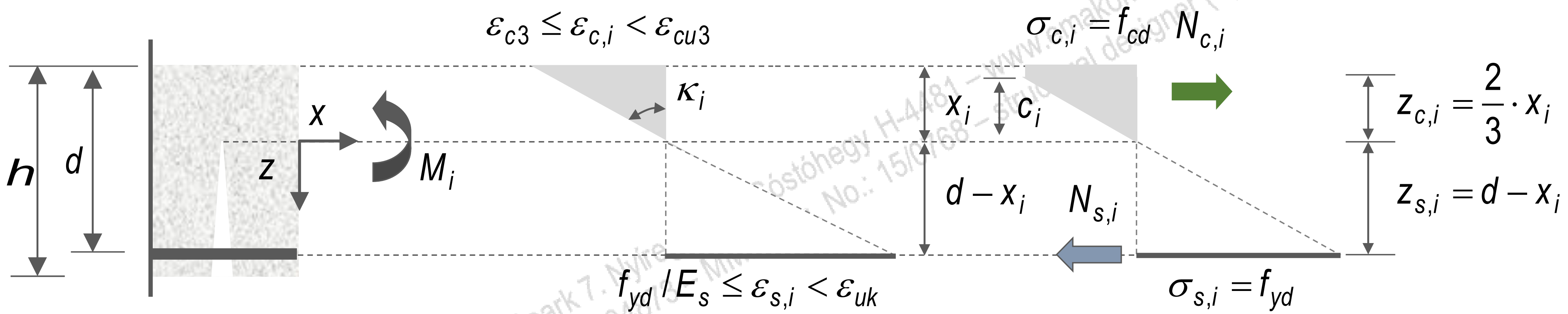
$\varepsilon_{c,i} [‰]$	x_i [mm]	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	M_i [kNm]	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i} [‰]$
ε_{c3} →	→	→	→	→ $\varepsilon_{s,II}$
…				…
….				…

Compressed concrete belt is already crushing, reinforcement yielding occurs



Compressed concrete belt is already crushing, reinforcement yielding occurs

Geometry | Deformation, ε | Normal stress, σ



$\varepsilon_{c,i}$ [‰]	x_i [mm]	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	M_i [kNm]	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i}$ [‰]
$\varepsilon_{c,i}$ ←				f_{yd}/E_s
…				…
….				…

Compressed concrete belt failure, reinforcement is yielding

The first plastic phenomenon is the **yielding of the reinforcement** → Tensioned reinforcement is yielding while the compressed concrete belt remains in the elastic state of stress → **Rupture of the tensioned reinforcement** occurs while the compressed concrete belt remains in the elastic state of stress ← **Typical failure mode of the under-reinforced RC cross-sections**

Tensioned reinforcement is yielding while the compressed concrete belt remains in the elastic state of stress → Tensioned reinforcement is yielding while plastic deformation (crushing) of the compressed concrete belt occurs

…the previously explained process can end with the failure of the compressed concrete belt, at which point the tensioned reinforcement is already yealding…

Tensioned reinforcement is yielding while plastic deformation (crushing) of the compressed concrete belt occurs → **Failure of the compressed concrete belt** occurs while the **tensioned reinforcement is yielding**

Tensioned reinforcement is yielding while plastic deformation (crushing) of the compressed concrete belt occurs → **Rupture of the tensioned reinforcement** occurs while the **compressed concrete belt is crushing**

Compressed concrete belt is crushing while plastic deformation (yielding) of the tensioned reinforcement occurs → **Failure of the compressed concrete belt** occurs while the **tensioned reinforcement is yielding**

Compressed concrete belt is crushing while plastic deformation (yielding) of the tensioned reinforcement occurs → **Rupture of the tensioned reinforcement** occurs while the **compressed concrete belt is crushing**

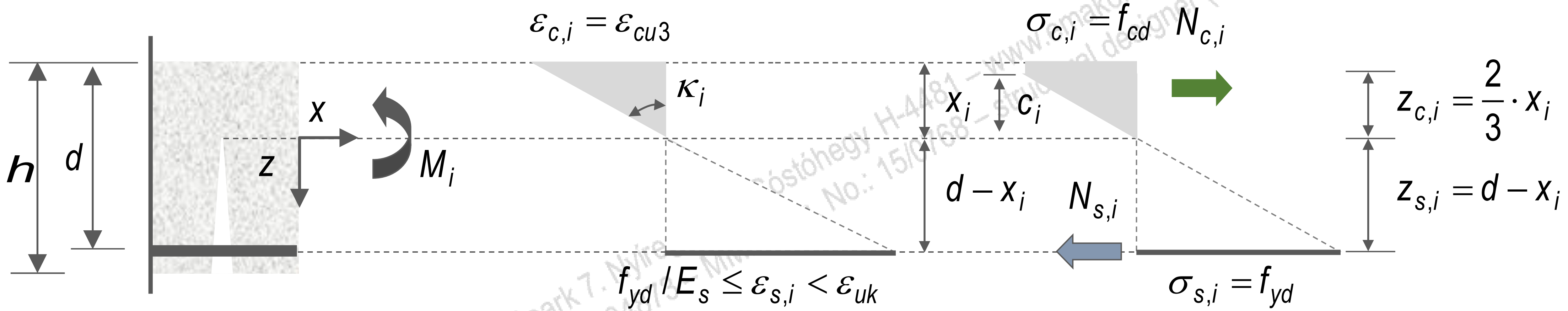
Typical failure modes of the normally-reinforced RC cross-sections!

The first plastic phenomenon is the **crushing of the concrete** → Compressed concrete belt is crushing while the tensioned reinforcement remains in the elastic state of stress → **Failure of the compressed concrete belt** occurs while the tensioned reinforcement remains in the elastic state of stress ← **Typical failure mode of the over-reinforced RC cross-sections**

Compressed concrete belt is crushing while the tensioned reinforcement remains in the elastic state of stress → Compressed concrete belt is crushing while plastic deformation (yielding) of the tensioned reinforcement occurs

Compressed concrete belt failure, reinforcement is yielding

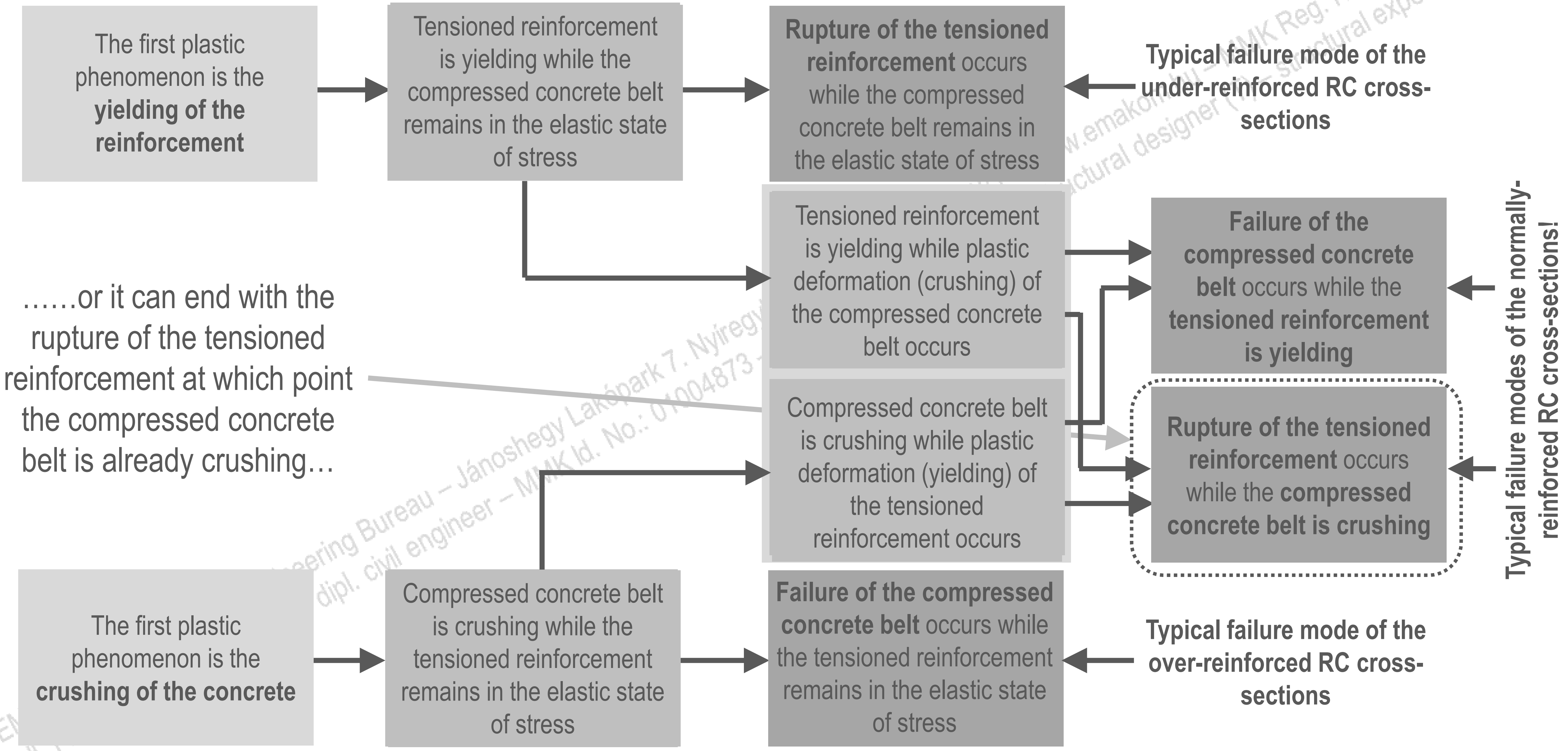
Geometry	Deformation, ε	Normal stress, σ



$\varepsilon_{c,i}$ [‰]	x_i [mm]	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	M_i [kNm]	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i}$ [‰]
…				…
$\varepsilon_{c,i}$				Controll for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c,i} = \varepsilon_{cu3}$!!!				$f_{yd}/E_s \le \varepsilon_{s,i} < \varepsilon_{uk}$

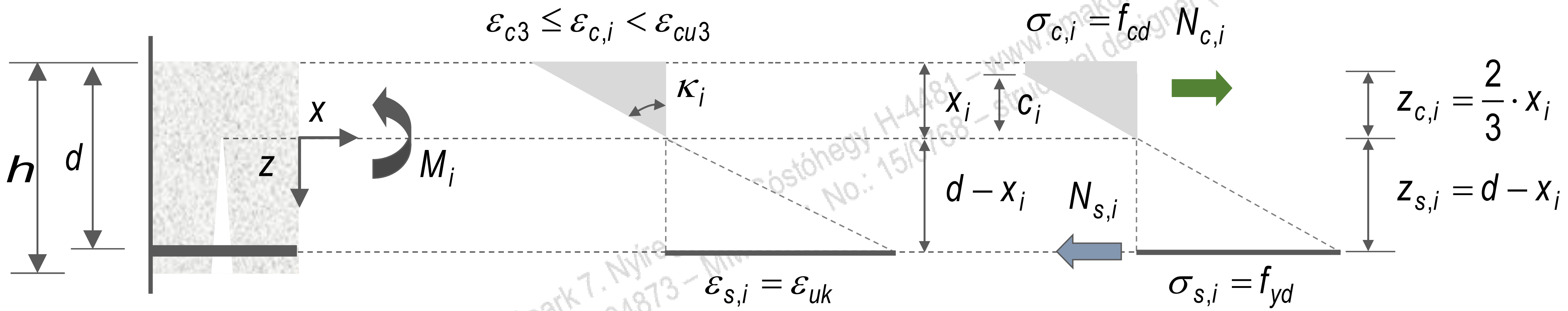
One of the typical failure modes of the normally-reinforced RC cross-sections!

Rupture of the reinforcement, compressed concrete belt is crushing



Rupture of the reinforcement, compressed concrete belt is crushing

Geometry	Deformation, ε	Normal stress, σ



$\varepsilon_{c,i} [‰]$	$x_i [mm]$	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i} \left[\frac{1}{mm}\right]$	$M_i [kNm]$	$\varepsilon_{s,i} = \varepsilon_{c,i} \frac{d - x_i}{x_i} [‰]$
…				…
$\varepsilon_{c,i}$				Controll for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c3} \le \varepsilon_{c,i} < \varepsilon_{cu3}$				$\varepsilon_{s,i} = \varepsilon_{uk}$!!!

Other typical failure mode of the normally-reinforced RC cross-sections!

End point of the intermediate state is no more than the failure – III. state of stress

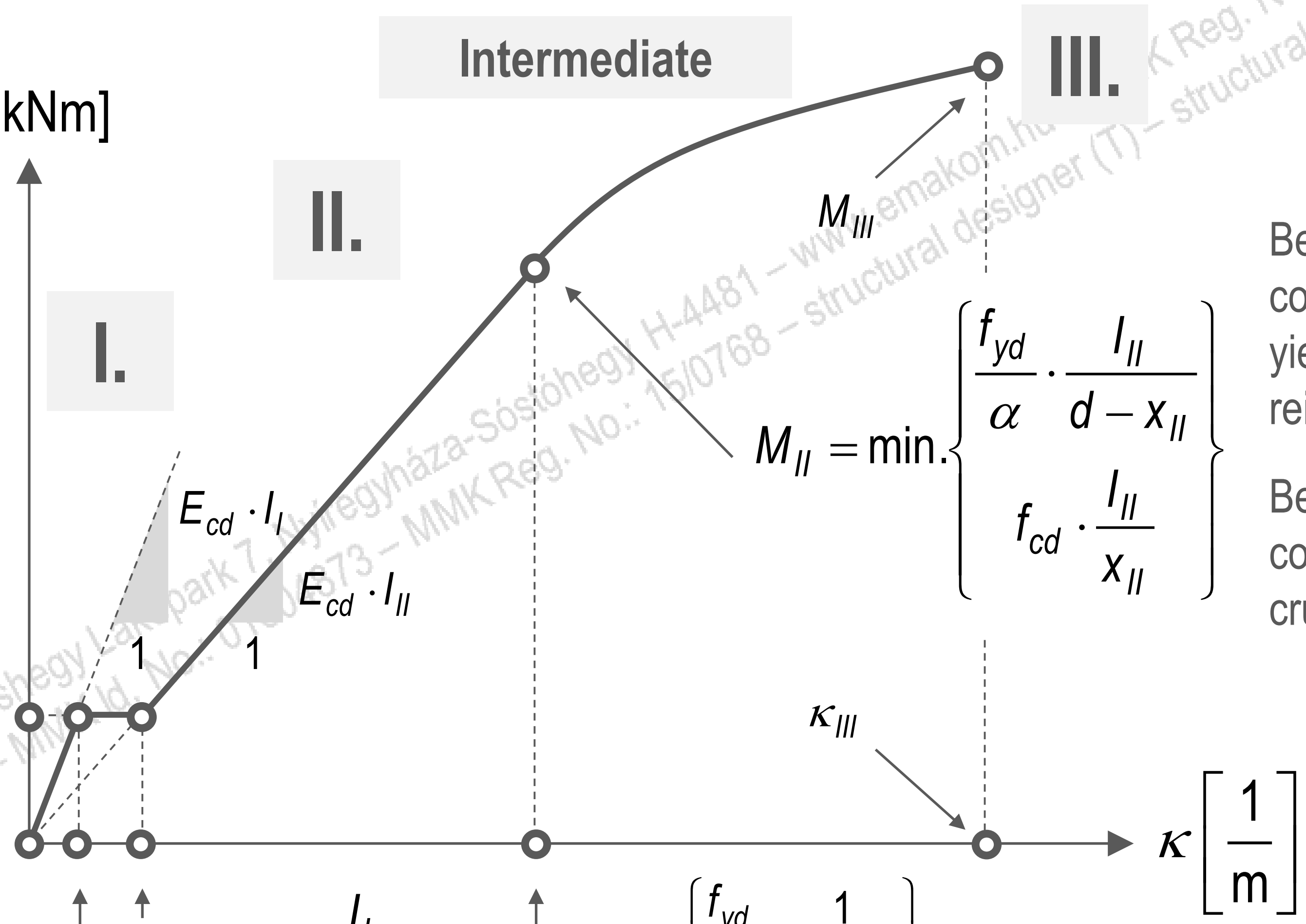
$\varepsilon_{c,i}\left[‰\right]$	$x_i\left[mm\right]$	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i}\left[\frac{1}{mm}\right]$	$M_i\left[kNm\right]$	$\varepsilon_{s,i} = \varepsilon_{c,i}\frac{d-x_i}{x_i}\left[‰\right]$
…				…
$\varepsilon_{c,i}$				Controll for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c,i} = \varepsilon_{cu3}$!!!				$f_{yd}/E_s \le \varepsilon_{s,i} < \varepsilon_{uk}$

One of the typical failure modes of the normally-reinforced RC cross-sections!

$\varepsilon_{c,i}\left[‰\right]$	$x_i\left[mm\right]$	$\kappa_i = \frac{\varepsilon_{c,i}}{x_i}\left[\frac{1}{mm}\right]$	$M_i\left[kNm\right]$	$\varepsilon_{s,i} = \varepsilon_{c,i}\frac{d-x_i}{x_i}\left[‰\right]$
…				…
$\varepsilon_{c,i}$				Controll for $\varepsilon_{s,i}$ value!
….				…
$\varepsilon_{c3} \le \varepsilon_{c,i} < \varepsilon_{cu3}$				$\varepsilon_{s,i} = \varepsilon_{uk}$!!!

Other typical failure mode of the normally-reinforced RC cross-sections!

The I., the II., the intermediate and the III. state of stresses



$$M_{II} = \min.\begin{cases} \dfrac{f_{yd}}{\alpha} \cdot \dfrac{I_{II}}{d - x_{II}} \\ f_{cd} \cdot \dfrac{I_{II}}{x_{II}} \end{cases}$$

Bending moment corresponds to the yielding of reinforcement

Bending moment corresponds to the crushing of concrete

$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I}$$

$$M_I = M_{I-II} \rightarrow \kappa_I \cdot E_{cd} \cdot I_I = \kappa_{I-II} \cdot E_{cd} \cdot I_{II} \rightarrow \kappa_{I-II} = \kappa_I \cdot \frac{I_I}{I_{II}}$$

$$\kappa_I = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h - x_I}$$

$$\kappa_{II} = \min.\begin{cases} \dfrac{f_{yd}}{E_s} \cdot \dfrac{1}{d - x_{II}} \\ \dfrac{f_{cd}}{E_{cd}} \cdot \dfrac{1}{x_{II}} \end{cases}$$

Curveture corresponds to the yielding of reinforcement

Curveture corresponds to the crushing of concrete

Application of the I., the II., the intermediate and the III. state of stresses

Ultimate limit state **ULS**

$$p_{Ed} = \gamma_G \cdot G_k + \gamma_Q \cdot Q_k \rightarrow M_{Ed} \le M_{Rd}$$

$$\text{e.g}: \; p_{Ed} = 1{,}35 \cdot 8{,}60 + 1{,}50 \cdot 2{,}50 \approx 15{,}40 \text{ kN/m}^2$$

Serviceability limit state **SLS**

$$p_{E,char} = G_k + Q_k \rightarrow M_{E,char}$$

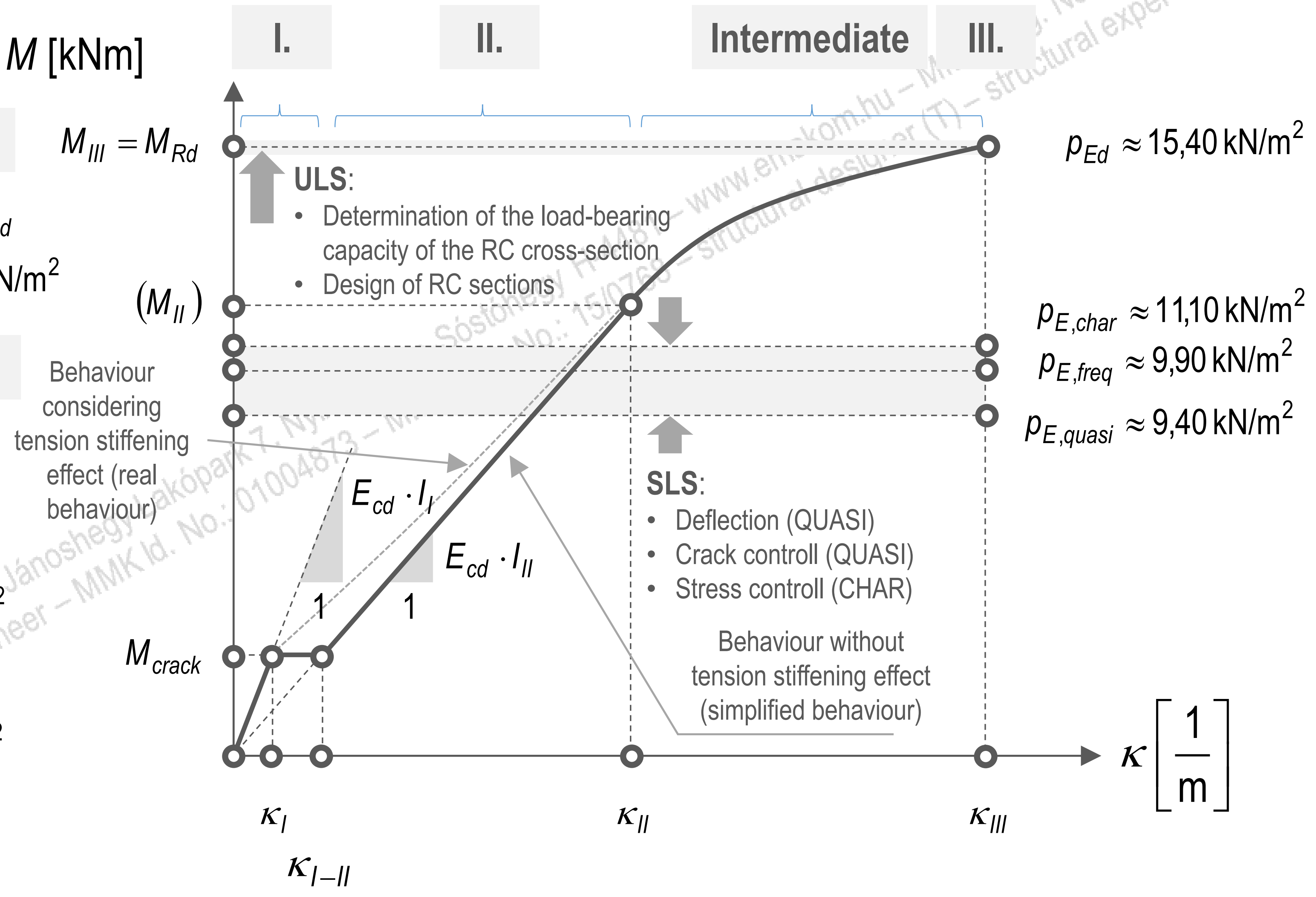
$$\text{e.g}: \; p_{E,char} = 8{,}60 + 2{,}50 \approx 11{,}10 \text{ kN/m}^2$$

$$p_{E,freq} = G_k + \psi_1 \cdot Q_k \rightarrow M_{E,freq}$$

$$\text{e.g}: \; p_{E,freq} = 8{,}60 + 0{,}50 \cdot 2{,}50 \approx 9{,}90 \text{ kN/m}^2$$

$$p_{E,quasi} = G_k + \psi_2 \cdot Q_k \rightarrow M_{E,quasi}$$

$$\text{e.g}: \; p_{E,quasi} = 8{,}60 + 0{,}30 \cdot 2{,}50 \approx 9{,}40 \text{ kN/m}^2$$



Reinforced Concrete (RC) Structures

Topic 15.

Intermediate state of stress

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Thank you for your kind attantion!