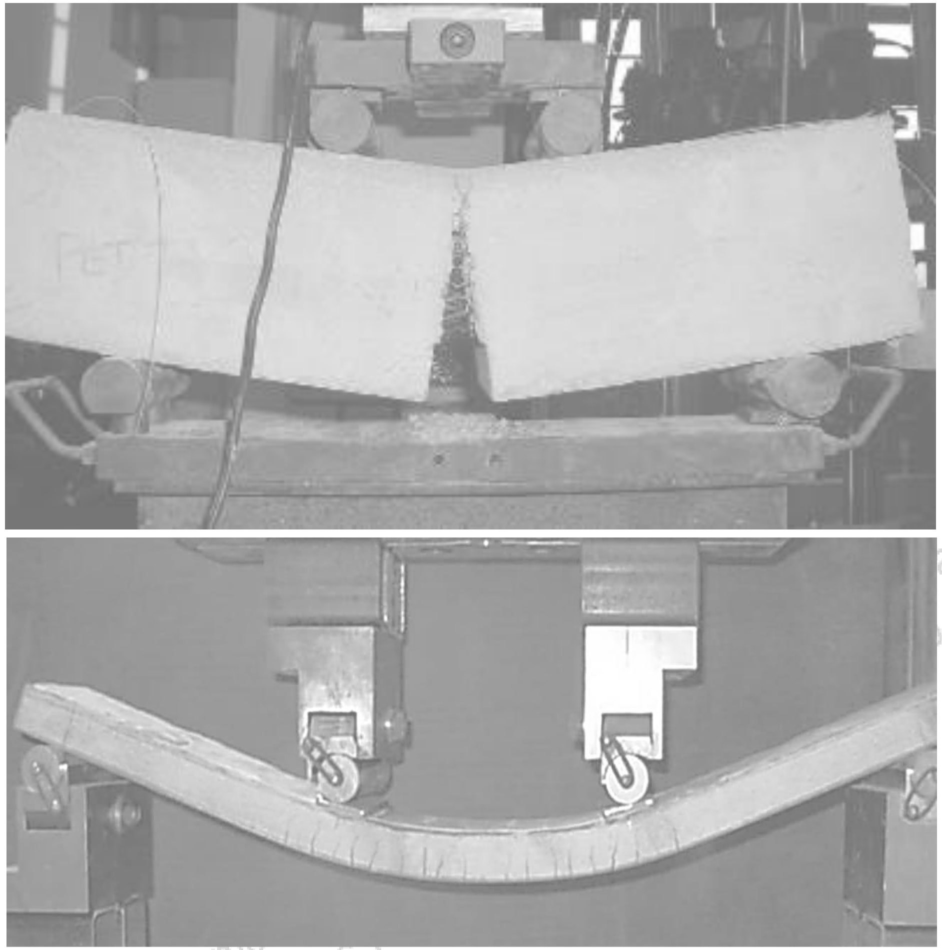

Reinforced Concrete (RC) Structures

Topic 5.

Strain properties of structural concrete

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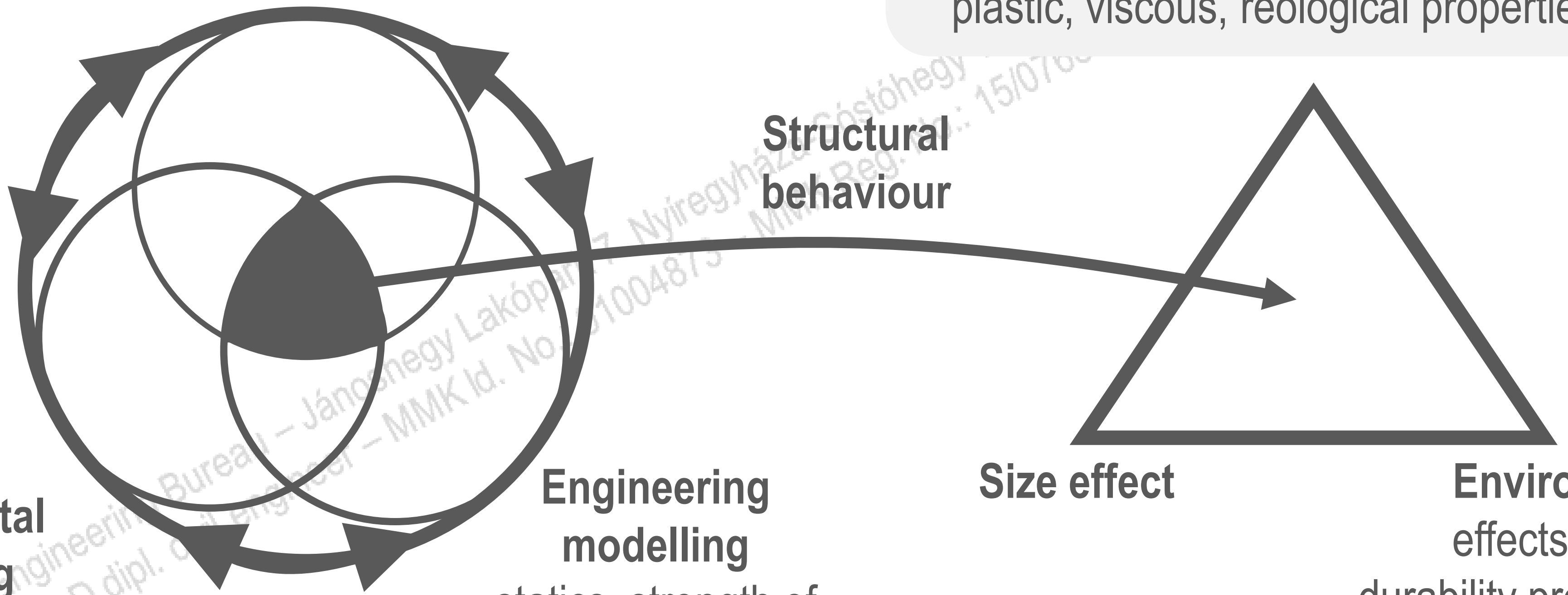
Modeling of structural behaviour of RC members



Numerical modelling
linear, non linear, plastic, 1st order
theory, 2nd order theory, etc.

Material properties
homogen, inhomogen, izotropic, anizotropic
elastic linear, non linear elastic,
plastic, viscous, reological properties, etc.

Structural behaviour

Experimental modelling
full-scale,
non full-scale, etc.

Engineering modelling
statics, strength of
materials, elasticity,
plasticity, dynamics, etc.

Size effect

Environment
effects, loads,
durability properties, etc.

Strain properties of structural concrete – *fib* Bulletin 51

$$\varepsilon_c(t) = \underbrace{\varepsilon_c(t_0) + \varepsilon_{cc}(t,t_0)}_{\text{total stress-dependent concrete strain at a concrete age } t} + \underbrace{\varepsilon_{cs}(t) + \varepsilon_{cT}(t,T)}_{\text{total stress independent concrete strain at a concrete age } t}$$

time-dependent concrete **strain**

time independent concrete **strains**

$\varepsilon_c(t)$ ➡ is the **total concrete strain** at age ***t***

$\varepsilon_c(t_0)$ ➡ is the stress-dependent **initial elastic strain** of concrete at time of stress application t_0

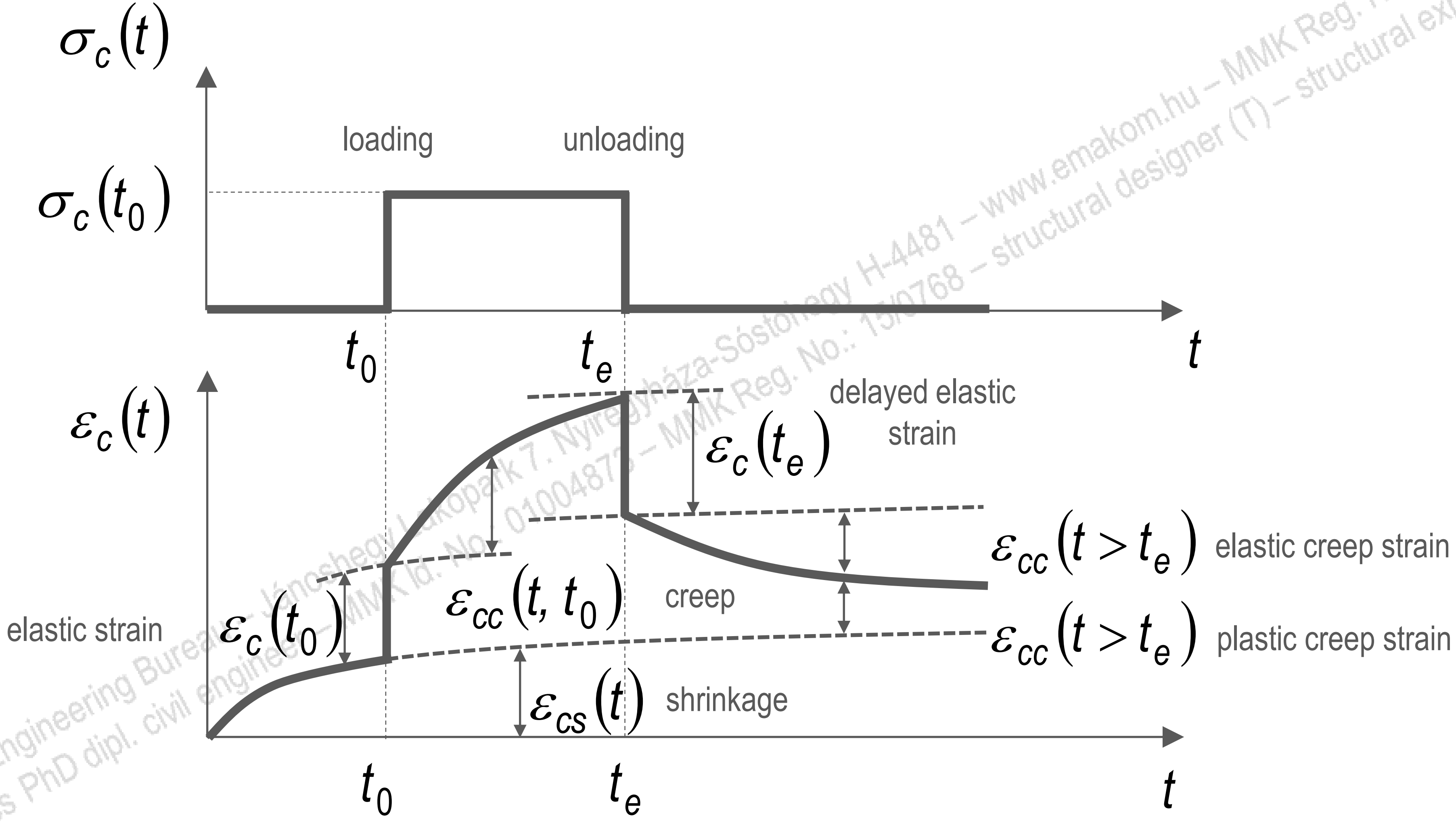
$\varepsilon_{cc}(t,t_0)$ ➡ is the **creep strain** of concrete at a concrete age $t \geq t_0$

$\varepsilon_{cs}(t)$ ➡ is the **shrinkage strain** of concrete at a concrete age ***t***

$\varepsilon_{cT}(t,T)$ ➡ is the **thermal strain** of concrete at a concrete age ***t***

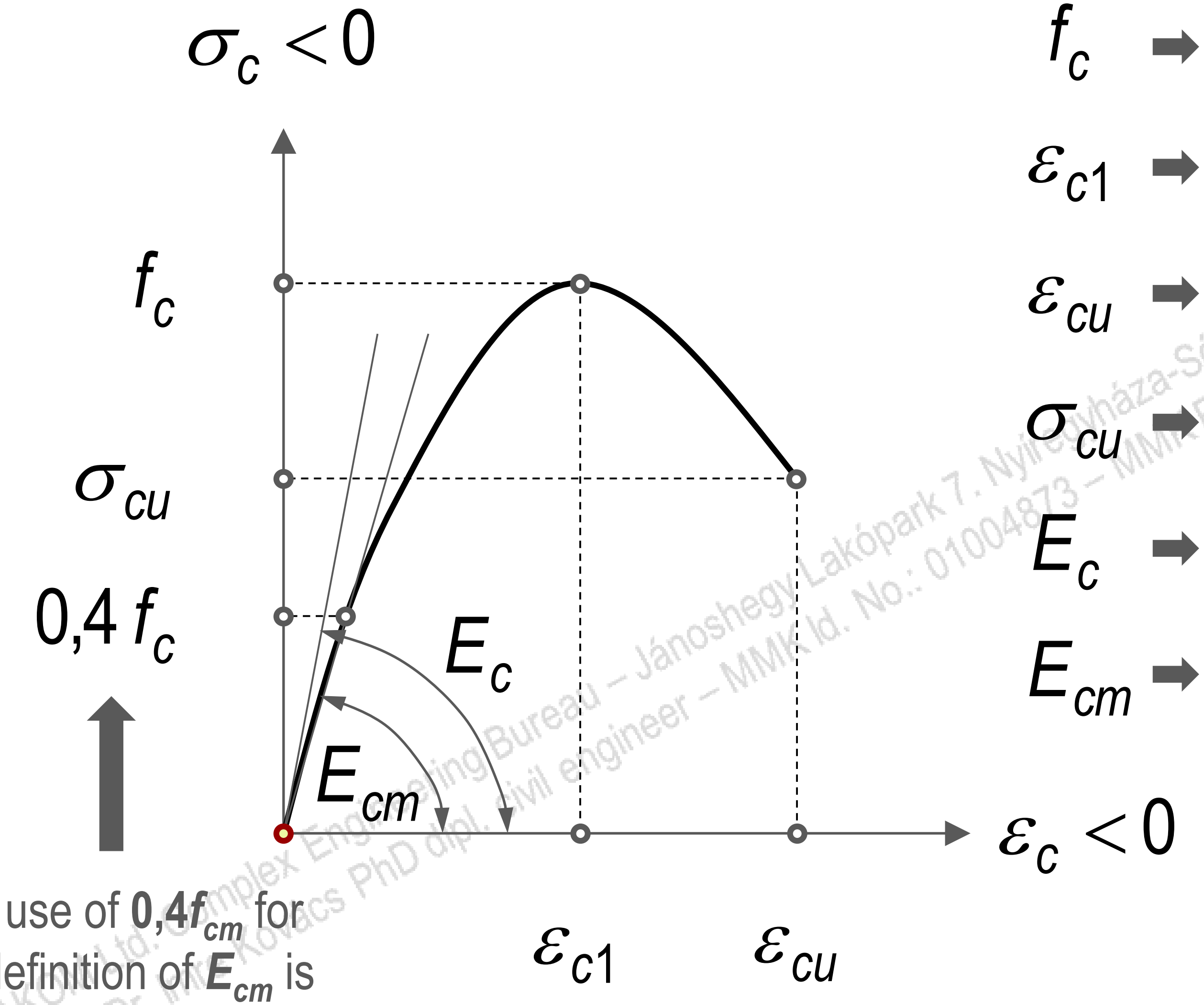
[fib Bulletin 51 – Structural Concrete – Volume 1. – Section 3.1.6 – Page 53.-72.]

Strain properties of structural concrete – *fib* Bulletin 51



[fib Bulletin 51 – Structural Concrete – Volume 1. – Fig. 3.1-15 – Page 57.]

Stress-strain relation for structural analysis – MSZ EN 1992-1-1:2010



The use of **0,4f_{cm}** for the definition of **E_{cm}** is approximate.

f_c ➡ is the peak value of the compressive stress of concrete

ε_{c1} ➡ is the concrete strain at peak stress

ε_{cu} ➡ is the ultimate compressive strain

σ_{cu} ➡ is the concrete stress at ultimate compressive strain

E_c ➡ is the tangent modulus of elasticity of concrete at $\sigma_c = 0$

E_{cm} ➡ is the secant modulus of elasticity of concrete

[according to MSZ EN 1992-1-1:2010 – Section 3.1.5 – Figure 3.2 – Page 34.]

Modulus of elasticity of structural concrete – MSZ EN 1992-1-1:2010

The elastic deformations of concrete **largely depend on its composition** (especially the aggregates). The values given in **MSZ 1992-1-1:2010** should be regarded as indicaive for general applications. However, they should be specifically assessed if the structure is likely to be sensitive to deviations from these general values.

[MSZ EN 1992-1-1:2010 – Section 3.1.3 – Paragraph (1) – Page 28.]

Modulus of elasticity of normal strength, normal density concretes: C12/15…C50/60

[kN/mm^2]	C12/15	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60
E_{cm}	27	29	30	31	33	34	35	36	37

and …C55/67…C90/105

[kN/mm^2]	C55/67	C60/75	C70/85	C80/95	C90/105
E_{cm}	38	39	41	42	44

$$E_{cm} = 22 \cdot \left(\frac{f_{cm}}{10} \right)^{0,3}$$

[according to MSZ EN 1992-1-1:2010 – Table 3.1 – Page 29.]

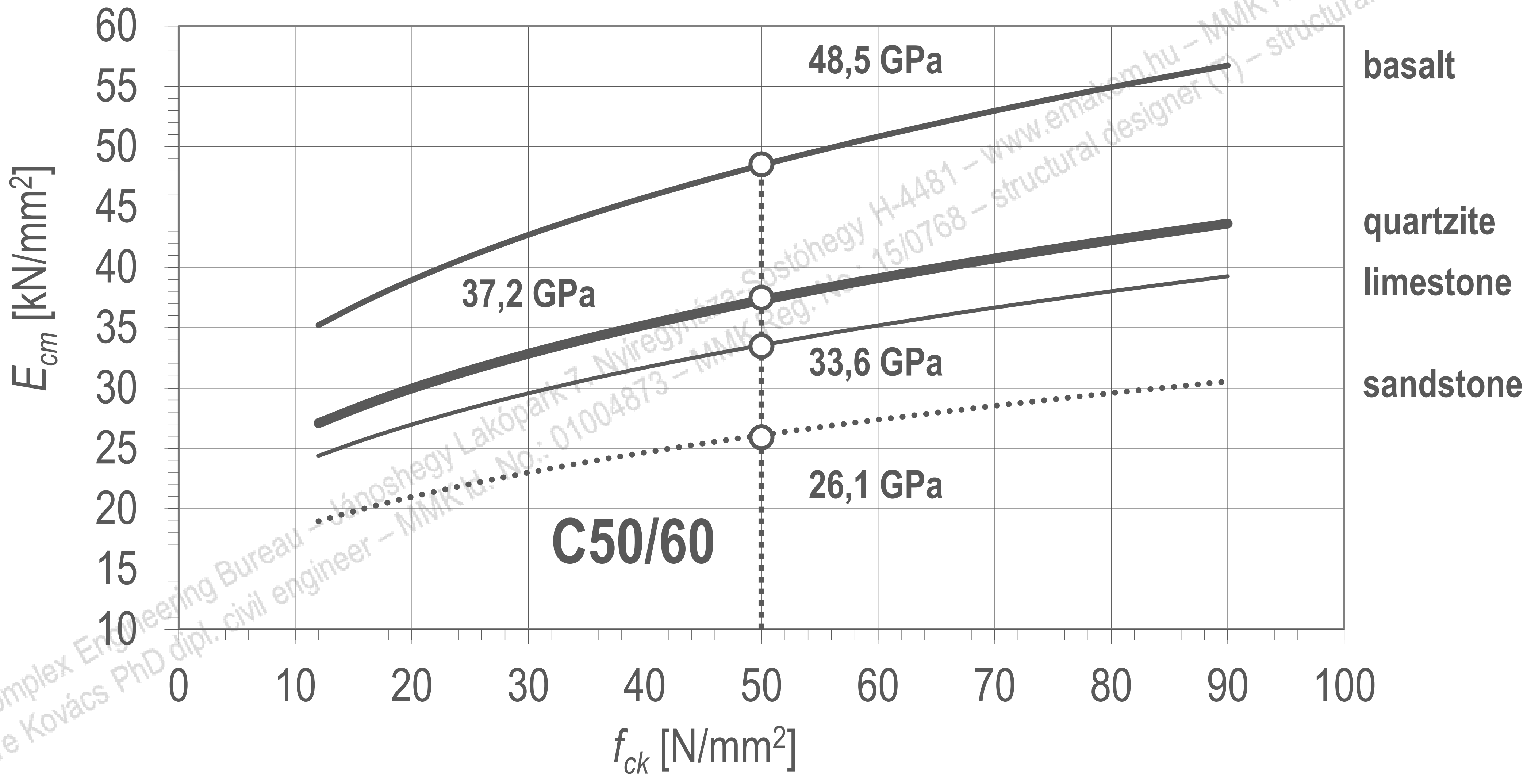
Effect of aggregate type on the modulus of elasticity – MSZ EN 1992-1-1:2010

The **modulus of elasticity of a concrete is controlled by the moduli of elasticity of its components**. Approximate values for the modulus of elasticity Ecm, secant value between sc = 0 and 0,4 fcm for concretes with **quartzite aggregates**, are given in **MSZ 1992-1-1:2010 Table 3.1**. For **limestone** and **sandstone aggregates** the value **should be reduced by 10%** and **30%** respectively. For **basalt aggregates** the value **should be increased by 20%.**

[MSZ EN 1992-1-1:2010 – Section 3.1.3 – Paragraph (2) – Page 28.]

Quartzite aggregate:	E_{cm}	**100%**
Limestone aggregate:	E_{cm}	**90%**
Sandstone aggregate:	E_{cm}	**70%**
Basalt aggregate:	E_{cm}	**120%**

Effect of aggregate type on the modulus of elasticity – MSZ EN 1992-1-1:2010



Variation of the modulus of elasticity with time – MSZ EN 1992-1-1:2010

Variation of the **modulus of elasticity** with time can be **estimated** by:

$$E_{cm}(t)=\left(\frac{f_{cm}(t)}{f_{cm}}\right)^{0,3}\cdot E_{cm}$$

⬆

It can be calculated by the relation of Arrhenius

⬇

$$E_{cm}(t)=\left[\beta_{cc}(t)\right]^{0,3}\cdot E_{cm}$$

$E_{cm}(t)$ ➡ is the secant modulus of elasticity of concrete at a concrete **age *t***

$f_{cm}(t)$ ➡ is the mean value of the compressive strength of concrete at a concrete **age *t***

f_{cm} ➡ is the mean value ot the compressive strength of concrete at a concrete age **28 days**

E_{cm} ➡ Is the secant modulus of elasticity of concrete at a concrete age **28 days**

t ➡ is the age of the concrete in days

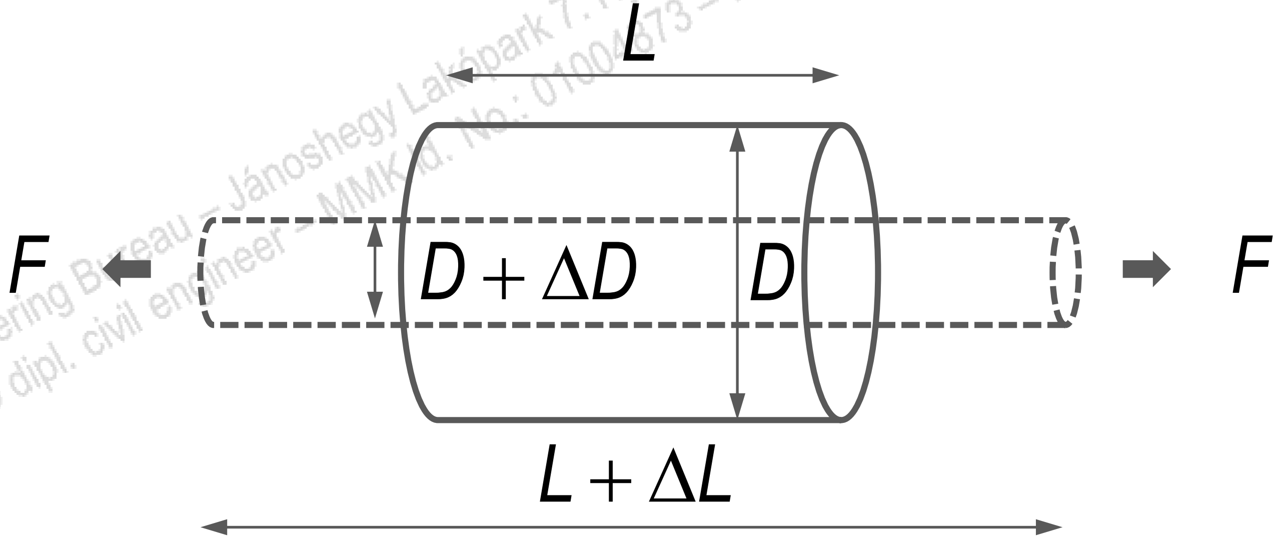
[MSZ EN 1992-1-1:2010 – Section 3.1.3 – Paragraph (3) – Page 30.]

Poisson’s ratio of structural concrete – MSZ EN 1992-1-1:2010

Poisson’s ration may be taken equal to **0,2** for **uncracked concrete** and **0** for **cracked concrete**.

[MSZ EN 1992-1-1:2010 – Section 3.1.3 – Paragraph (4) – Page 30.]

Poisson’s ratio: $$\nu = -\frac{\Delta L / L}{\Delta D / D}$$



Creep of structural concrete – *fib* Bulletin 51

$$\varepsilon_c(t) = \underbrace{\varepsilon_c(t_0) + \varepsilon_{cc}(t,t_0)} + \varepsilon_{cs}(t) + \varepsilon_{cT}(t,T)$$

total **stress-dependent** concrete
strain at a concrete age ***t***

$$\varepsilon_\sigma(t) = \varepsilon_c(t_0) + \varepsilon_{cc}(t,t_0)$$

$\varepsilon_c(t)$ ➡ is the **total concrete strain** at age ***t***

$\varepsilon_c(t_0)$ ➡ is the stress-dependent **initial elastic strain** of concrete at time of stress application t_0

$\varepsilon_{cc}(t,t_0)$ ➡ is the **creep strain** of concrete at a concrete age $t \geq t_0$

$\varepsilon_{cs}(t)$ ➡ is the **shrinkage strain** of concrete at a concrete age ***t***

$\varepsilon_{cT}(t,T)$ ➡ is the **thermal strain** of concrete at a concrete age ***t***

[fib Bulletin 51 – Structural Concrete – Volume 1. – Section 3.1.6 – Page 53.-72.]

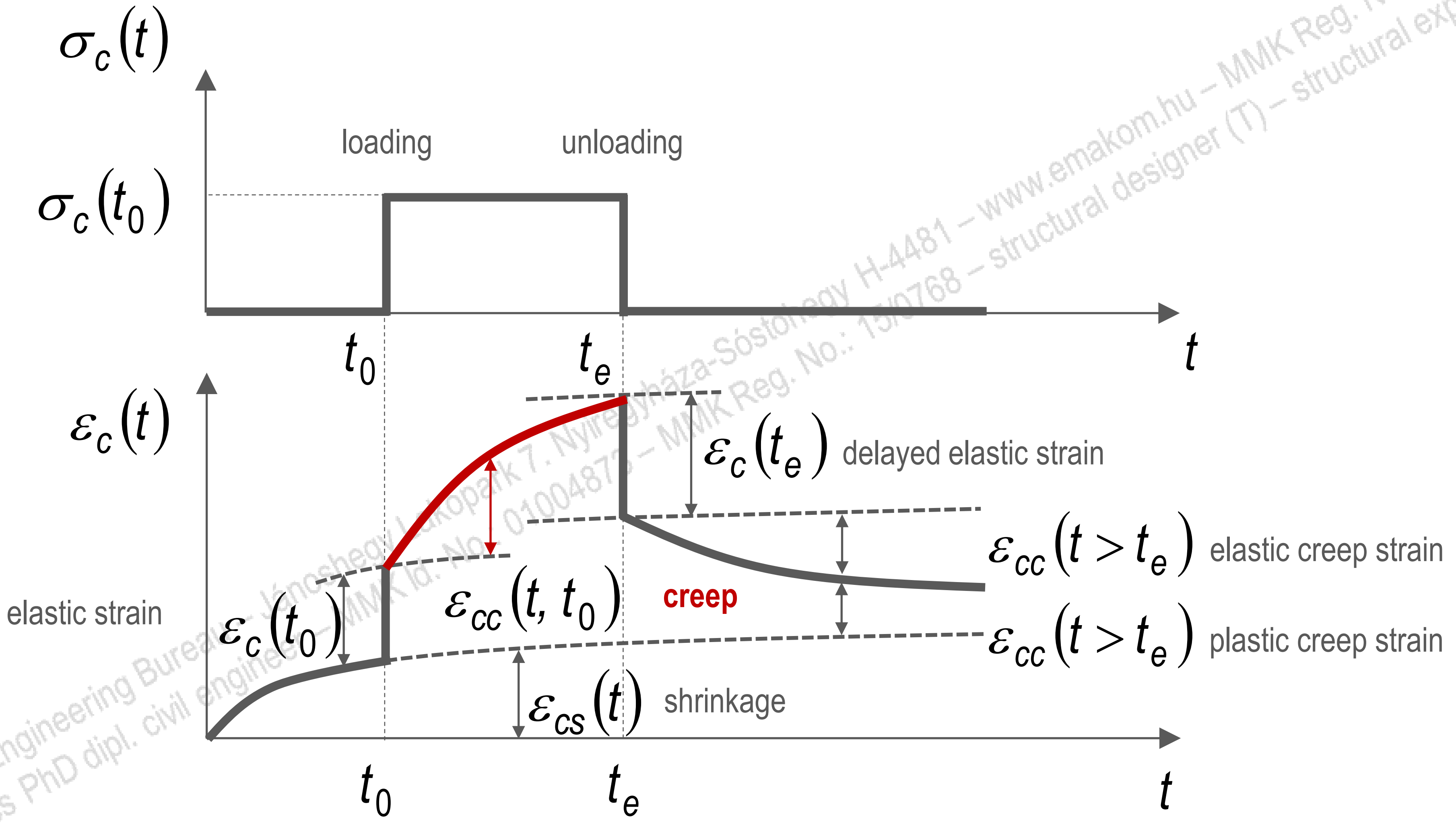
Creep of structural concrete – MSZ EN 1992-1-1:2010

Creep of the concrete depend on the:

- ambient **humidity**
  (environmental effect)
- **dimensions** of the element
  (size effect)
- **composition of concrete**
  (properties of components)
- **maturity** of the concrete when the **load is first applied**
  (early strengths, early modulus of elasticity of concrete, formworking, etc.)
- **duration** of the loading
  (short term loading, long term loading, etc.)
- **magnitude** of loading

[according to MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (1)P – Page 30.]

Creep of structural concrete – *fib* Bulletin 51



[fib Bulletin 51 – Structural Concrete – Volume 1. – Fig. 3.1-15 – Page 57.]

Creep of structural concrete – *fib* Bulletin 42

$$\varepsilon_\sigma(t)=\varepsilon_c(t_0)+\varepsilon_{cc}(t,t_0)$$

stress-dependent concrete strain at concrete age ***t***

stress-dependent initial elastic strain of concrete at time of stress application ***t₀***

creep strain of concrete at a concrete age ***t***

$$\varepsilon_\sigma(t)=\varepsilon_c(t_0)+\varepsilon_{cc}(t,t_0)=\sigma_c(t_0)\cdot J(t,t_0)$$

$$J(t,t_0)=\frac{1}{E_c(t_0)}+\frac{\varphi(t,t_0)}{E_c}$$

$$\varepsilon_c(t_0)=\frac{\sigma_c(t_0)}{E_c(t_0)}$$

$$\varepsilon_{cc}(t,t_0)=\varphi(t,t_0)\cdot\frac{\sigma_c(t_0)}{E_c}$$

$$\varepsilon_\sigma(t)=\varepsilon_c(t_0)+\varepsilon_{cc}(t,t_0)=\frac{\sigma_c(t_0)}{E_c(t_0)}+\varphi(t,t_0)\cdot\frac{\sigma_c(t_0)}{E_c}$$

[fib Bulletin 42 – Constitutive modelling of high strength/high performance concrete – Chapter 6.4.4 – pp. 78-79.]

Consequences of inadequate consideration of creep

Deformation of bridge
due to creep

Determination of creep coefficient – MSZ EN 1992-1-1:2010

The creep coefficient, $\phi(t,t_0)$ is related to E_c, the tangent modulus, which may be taken as $\mathbf{1{,}05\times E_{cm}}$. Where great accuracy is not required, tha value found from **MSZ EN 1992-1-1:2010 Table 3.1** may be considered as the creep coefficient, provided that the concrete is not subjected to a compressive stress greater than $\mathbf{0{,}45\times f_{ck}(t_0)}$ at an age t_0, the age of concrete at the time of loading.

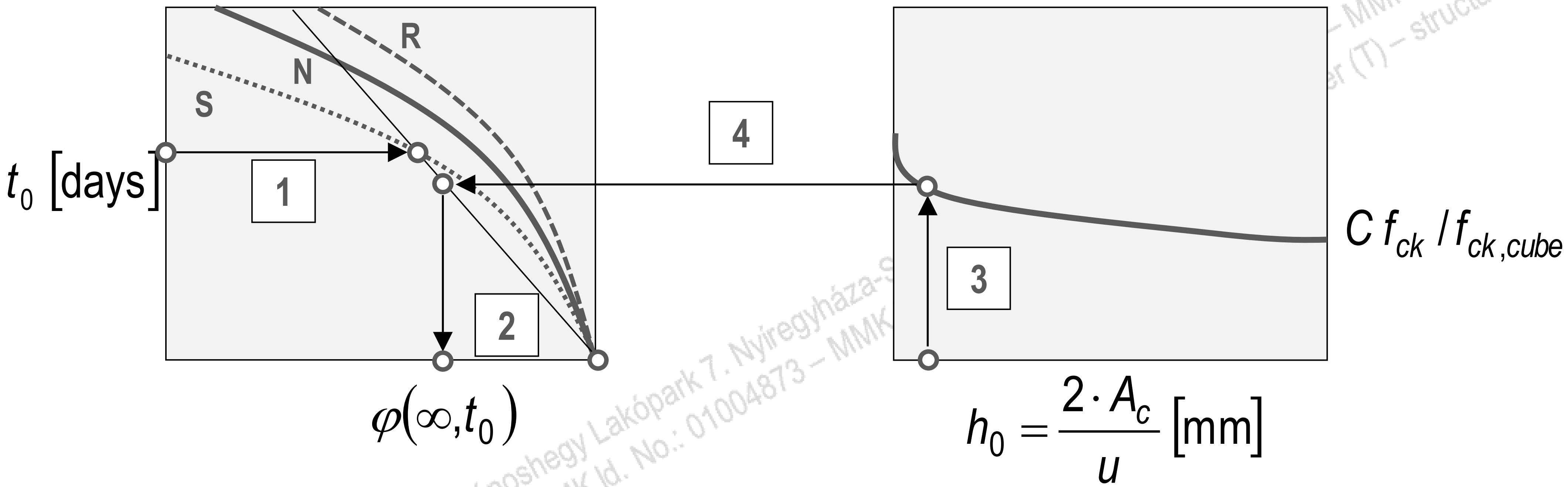
[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (2) – Page 30.]

The creep deformation of concrete $\varepsilon_{cc}(\infty,t_0)$ at time $t = \infty$ for a constant compressive stress σ_c applied at the concrete age t_0, is given by:

$$\varepsilon_{cc}(\infty,t_0) = \varphi(\infty,t_0)\cdot\frac{\sigma_c(t_0)}{E_c}$$

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (3) – Page 30.]

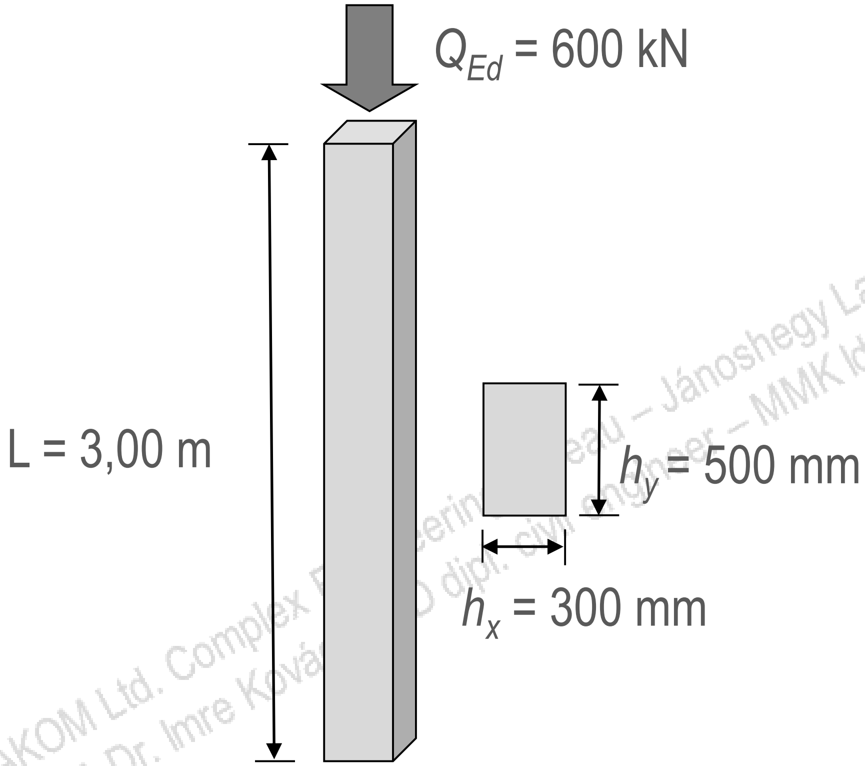
Determination of creep coefficient – MSZ EN 1992-1-1:2010



- **I.** Choice of environmental condition **(Inside RH=50%,Outside RH=80%)**
- **II.** Determination of the **age of loading** in days
- **III.** Choice of the **type of cement**
- **IV.** Choice of the **compressive strength class** of concrete
- **V.** Calculation of the **notional size**
- **VI.** Determination of the **final creep coefficient**

Example for the use of the final creep coefficient

Determine the deformation of a reinforced concrete column using the following cross section: **300/500 mm**, height: **L = 3000 mm**, strength class: **C20/25**, type of aggregate: **sandy gravel**, type of cement: **CEM I 52.5 R** under **inside conditions (RH = 50%)**, if the design value of the load **600 kN** will be applied at the **age of 7 days after concreting**!



Environmental condition:	**inside RH=50%**
Age of loading (formworking):	**t_0 = 7 days**
Type of cement:	**CEM I. 52,5 R** (**Class R**)
Concrete strength grade:	**C20/25**
Type of aggregates:	**sandy gravel**
Design value of the load:	**Q_{Ed} = 600 kN**

Example for the use of the final creep coefficient

Calculation of the **notional size**:

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 300 \cdot 500}{2 \cdot (300 + 500)} = \boxed{187{,}5\,\mathrm{mm}}$$

Elastic deformation of columne at a concrete age **7 days**:

$$\Delta L_{el}(t = 7\,\text{days}) = \frac{Q_{Ed} \cdot L}{E_c(t = 7\,\text{days}) \cdot A} = \frac{Q_{Ed} \cdot L}{1{,}05 \cdot E_{cm}(t = 7\,\text{days}) \cdot A}$$

➡ $$\beta_{cc}(t = 7\,\text{days}) = e^{s\left[1-\left(\frac{28}{t}\right)^{1/2}\right]} = e^{s\left[1-\left(\frac{28}{7}\right)^{1/2}\right]} = e^{0{,}2\left[1-\left\{\frac{28}{7}\right\}^{1/2}\right]} = e^{-0{,}2} = \boxed{0{,}82}$$

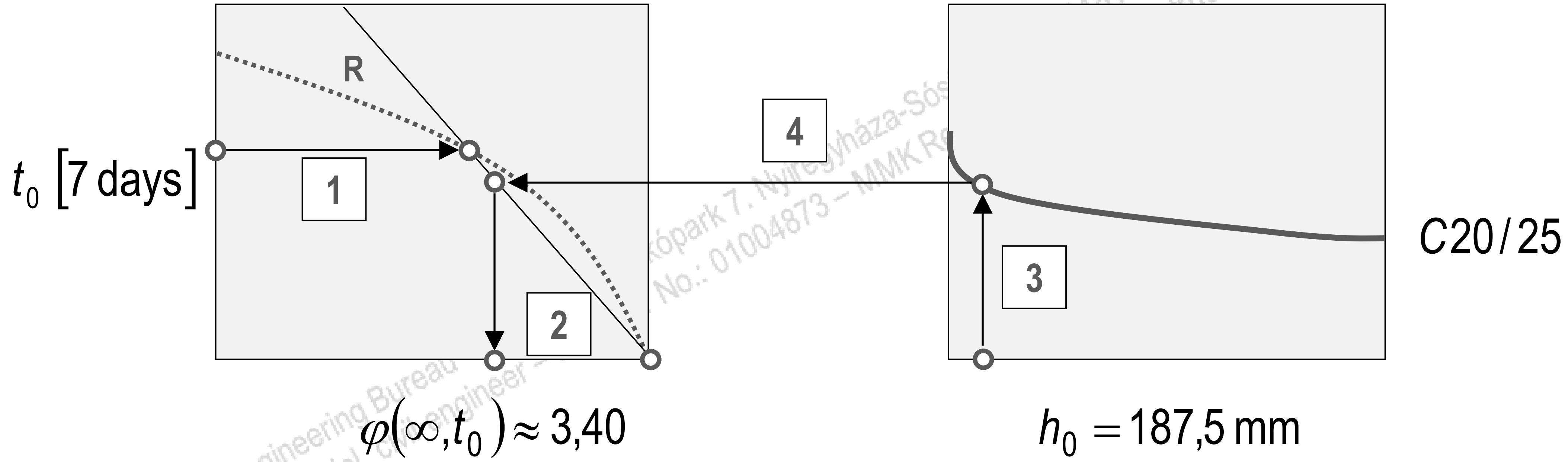
➡ $$f_{cm}(t = 7\,\text{days}) = \beta_{cc}(t = 7\,\text{days}) \cdot f_{cm} = 0{,}82 \cdot 28 = \boxed{23\,\mathrm{N/mm^2}}$$

➡ $$E_{cm}(t = 7\,\text{nap}) = E_{cm} \cdot \left(\frac{f_{cm}(t = 7\,\text{nap})}{f_{cm}}\right)^{0{,}3} = 30 \cdot \left(\frac{23}{28}\right)^{0{,}3} = \boxed{28{,}3\,\mathrm{GPa}}$$

$$\Delta L_{el}(t = 7\,\text{nap}) = \frac{Q_{Ed} \cdot L}{E_c(t = 7\,\text{nap}) \cdot A} = \frac{Q_{Ed} \cdot L}{1{,}05 \cdot E_{cm}(t = 7\,\text{nap}) \cdot A} = \frac{600 \cdot 10^3 \cdot 3000}{1{,}05 \cdot 28{,}3 \cdot 10^3 \cdot 500 \cdot 300} = \boxed{0{,}404\,\mathrm{mm}}$$

Example for the use of the final creep coefficient

Inside environmental condition: RH=50%



Example for the use of the final creep coefficient

Final creep coefficient of concrete:

$$\varphi(\infty,t_0)\approx\boxed{3{,}40}$$

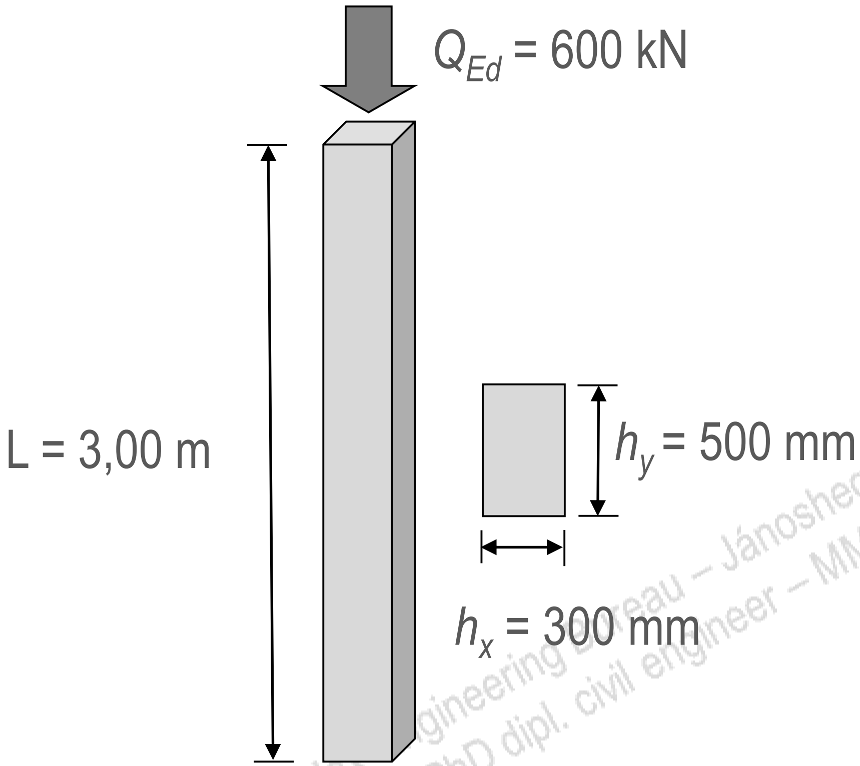
Final creep strain of concrete:

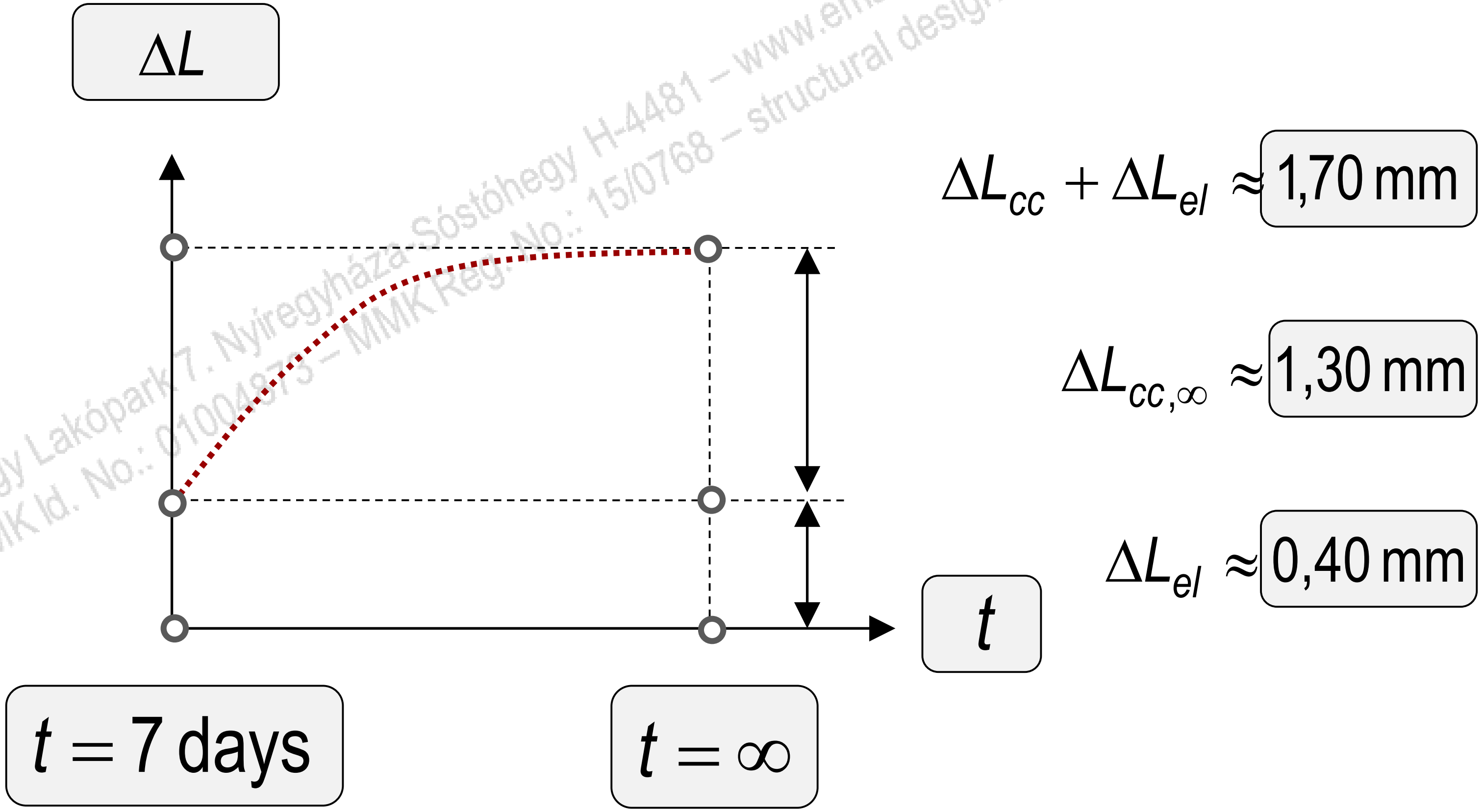
$$\varepsilon_{cc}(\infty,t_0)=\varphi(\infty,t_0)\cdot\frac{\sigma_c}{E_c}=\varphi(\infty,t_0)\cdot\frac{\sigma_c}{1{,}05\cdot E_{cm}}=3{,}40\cdot\frac{600\cdot10^3}{300\cdot500}\cdot\frac{1}{1{,}05\cdot30\cdot10^3}=\boxed{0{,}432\,‰}$$

Final value of the column deformation due to creep:

$$\Delta L_{cc,\infty}=\varepsilon_{cc}(\infty,t_0)\cdot L=0{,}432\,‰\cdot3000\text{ mm}=\boxed{1{,}296\text{ mm}}$$

Example for the use of the final creep coefficient





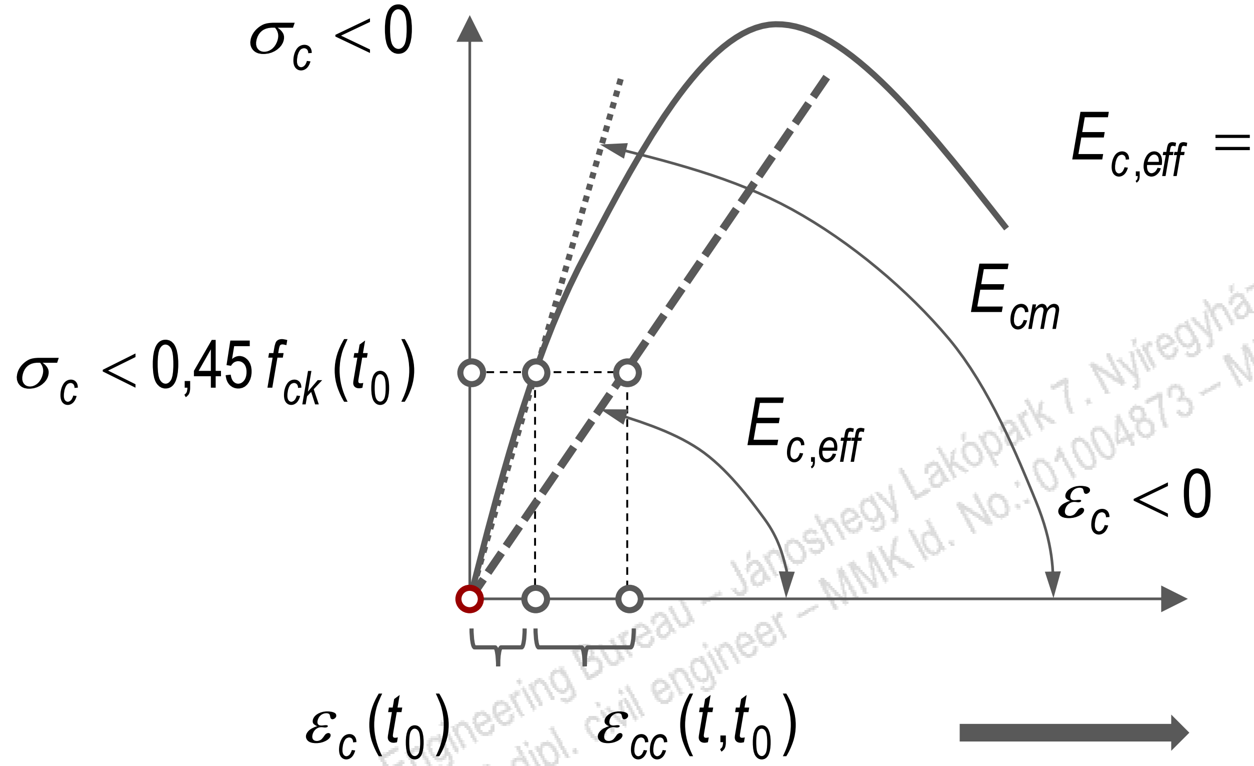
$\Delta L_{cc} + \Delta L_{el} \approx$ 1,70 mm

$\Delta L_{cc,\infty} \approx$ 1,30 mm

$\Delta L_{el} \approx$ 0,40 mm

Effective modulus of elasticity for concrete – MSZ EN 1992-1-1:2010

For loads with a duration causing creep, the **total deformation including creep** may be calculated by using an **effective modulus of elasticity for concrete** according to **MSZ EN 1992-1-1:2010 Equ. (7.20)**:



$$E_{c,eff} = \frac{E_{cm}}{1+\varphi(\infty,t_0)}$$ ➡ is the **effective modulus of elasticity for concrete**

$\varphi(\infty,t_0)$ ➡ is the **creep coefficient** relevant for the load tand time interval

E_{cm} ➡ is the **secant modulus of elasticity of concrete**

$$\varepsilon_\sigma(t) = \varepsilon_c(t_0) + \varepsilon_{cc}(t,t_0)$$

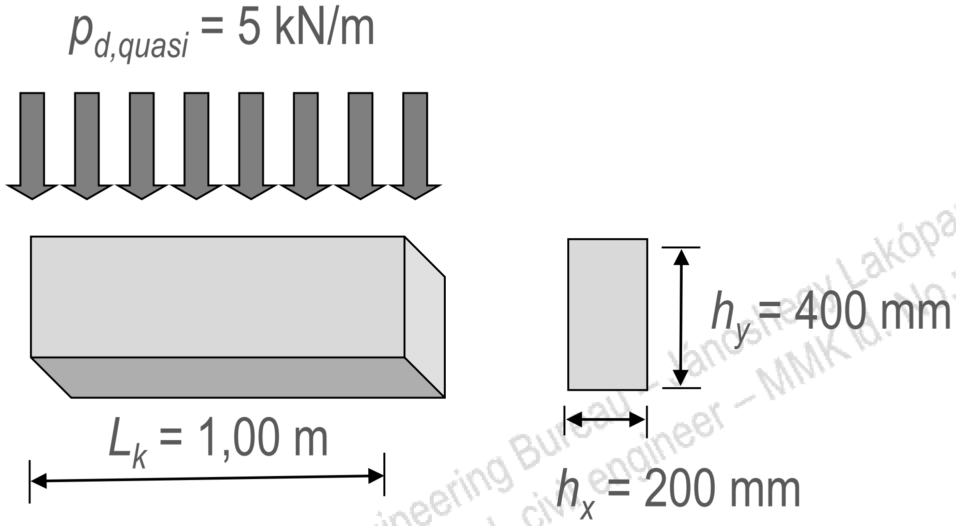
$\varepsilon_c(t_0)$ is the stress-dependent **initial elastic strain** of concrete at time of stress application ***t₀***

$\varepsilon_{cc}(t,t_0)$ is the **creep strain** of concrete at a concrete age ***t* ≥ *t₀***

$\varepsilon_\sigma(t)$ **stress-dependent** concrete strain at concrete age ***t***

Example for the use of the effective modulus of elasticity for concrete

Határozzuk meg az alábbi **200/400 mm** keresztmetszetű, **L = 1000 mm** méretű, **C20/25** szilárdsági osztályú, **homokkő adalékanyag** továbbá **CEM III 32,5 N cement** felhasználásával **kültéri viszonyok (RH = 80%)** között készülő **beton konzol** alakváltozását, ha a **5 kN/m** nagyságú (kvázi állandó) teher értékének felvitelére (kizsaluzás) a betonozást követően **3 nap**os korban kerül sor!



Environmental condition:	**Outside RH=80%**
Time of stress application t_0:	t_0 **= 3 days**
Type of cement:	**CEM III 32,5 N** (**Class S**)
Concrete grade:	**C20/25**
Type of aggregate:	**sandstone**
Quasi permanent load:	$p_{d,quasi}$ **= 5 kN/m**

Example for the use of the effective modulus of elasticity for concrete

Calculation of the **notional size**:

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 200 \cdot 400}{2 \cdot (200 + 400)} = \boxed{133{,}3\ \text{mm}}$$

Calculation of the **moment of inertia**:

$$I = \frac{b \cdot h^3}{12} = \frac{200 \cdot 400^3}{12} = \boxed{1{,}07 \cdot 10^9\ \text{mm}^4}$$

Determination of the **elastic deflection** at age **3 days**:

$$w_{el}(t = 3\ \text{days}) = \frac{1}{8} \cdot \frac{p_{d,quasi} \cdot {L_k}^4}{E_c(t = 3\ \text{days}) \cdot I} = \frac{p_{d,quasi} \cdot {L_k}^4}{1{,}05 \cdot E_{cm}(t = 3\ \text{days}) \cdot I}$$

➡ $$\beta_{cc}(t = 3\ \text{days}) = e^{s\left[1-\left(\frac{28}{t}\right)^{1/2}\right]} = e^{s\left[1-\left(\frac{28}{7}\right)^{1/2}\right]} = e^{0{,}38\left[1-\left\{\frac{28}{3}\right\}^{1/2}\right]} = e^{-0{,}78} = \boxed{0{,}46}$$

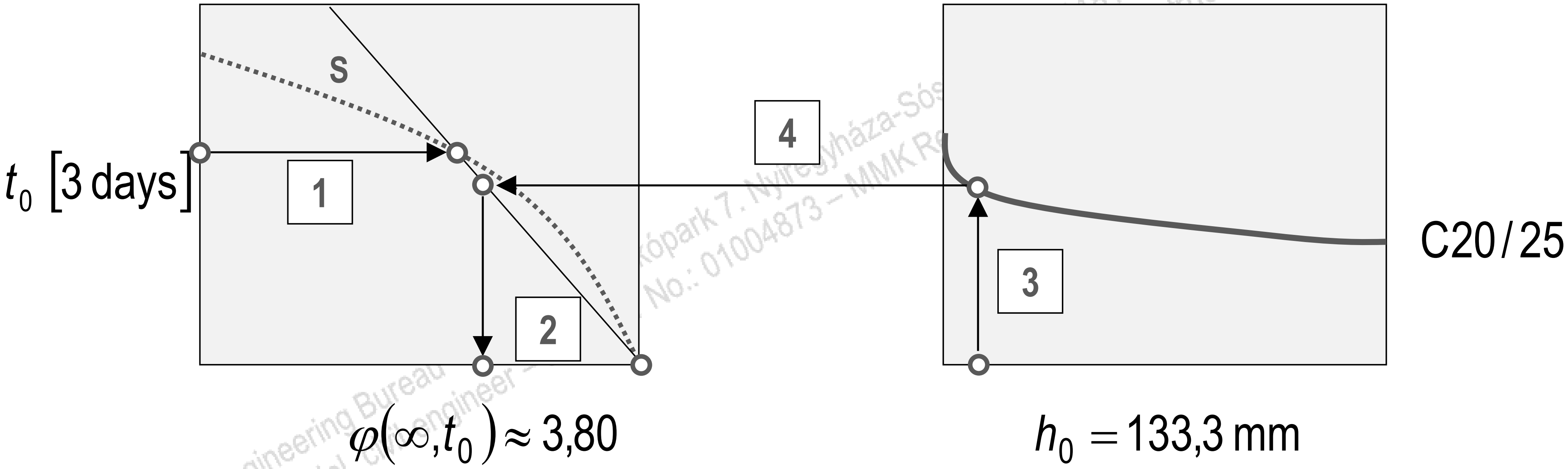
➡ $$f_{cm}(t = 3\ \text{days}) = \beta_{cc}(t = 3\ \text{days}) \cdot f_{cm} = 0{,}46 \cdot 28 = \boxed{12{,}9\ \text{N/mm}^2}$$

➡ $$E_{cm}(t = 3\ \text{nap}) = 0{,}70 \cdot E_{cm} \cdot \left(\frac{f_{cm}(t = 3\ \text{nap})}{f_{cm}}\right)^{0{,}3} = 0{,}70 \cdot 30 \cdot \left(\frac{12{,}9}{28}\right)^{0{,}3} = \boxed{16{,}7\ \text{GPa}}$$

$$w_{el}(t = 3\ \text{nap}) = \frac{1}{8} \cdot \frac{p_{d,quasi} \cdot {L_k}^4}{1{,}05 \cdot E_{cm}(t = 3\ \text{nap}) \cdot I} = \frac{1}{8} \cdot \frac{5 \cdot 1000^4}{1{,}05 \cdot 16700 \cdot 1{,}07 \cdot 10^9} = \boxed{0{,}033\ \text{mm}}$$

Example for the use of the effective modulus of elasticity for concrete

Environmental condition: Inside RH=80%



Example for the use of the effective modulus of elasticity for concrete

Final value of the **creep coefficient**:

$$\varphi(\infty,t_0)\approx\boxed{3{,}80}$$

Calculation of the **effective modulus of elasticity for concrete**:

$$E_{c,eff}=\frac{E_{cm}}{1+\varphi(\infty,t_0)}=\frac{30000}{1+3{,}80}=\boxed{6250\,\mathrm{N/mm^2}}$$

Calculation of the **final** value of teh **elastic deflection** (**elastic + creep**):

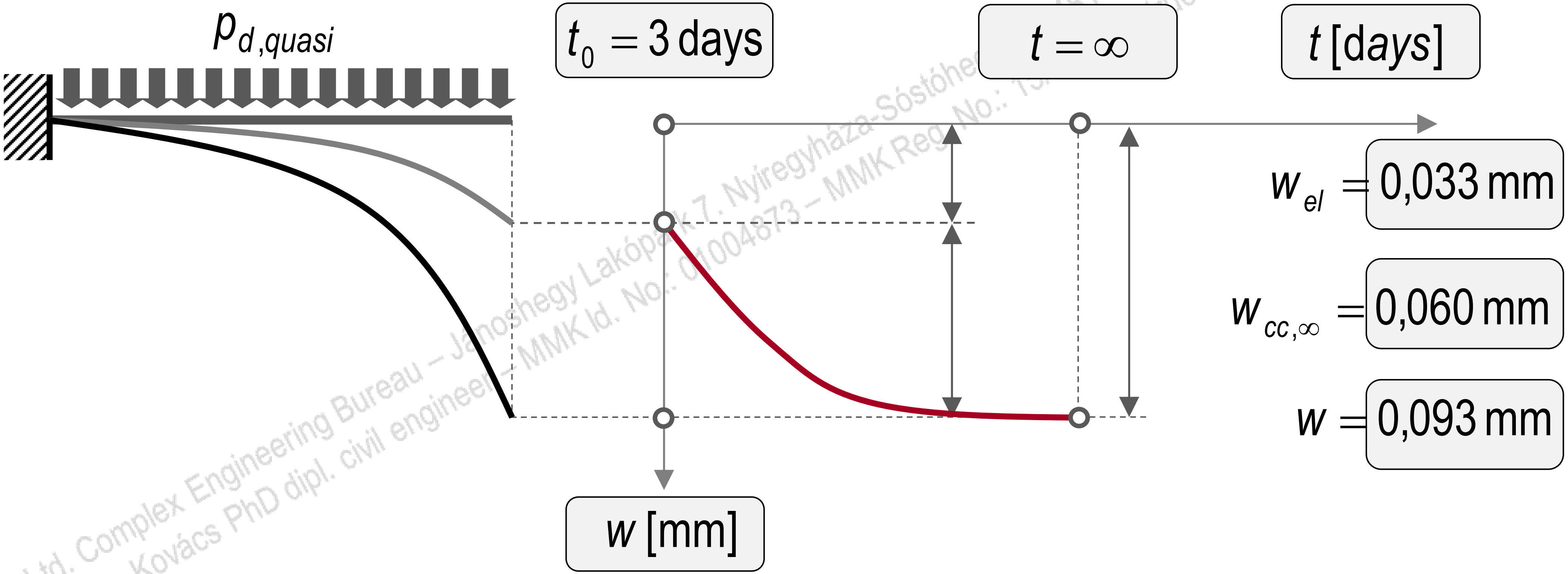
$$w(t=\infty)=w_{el}(t_0=3\,\text{days})+w_{cc,\infty}=\frac{1}{8}\cdot\frac{p_{d,quasi}\cdot L^4}{E_{c,eff}\cdot I}=\frac{1}{8}\cdot\frac{5\cdot1000^4}{6250\cdot1{,}07\cdot10^9}=\boxed{0{,}093\,\mathrm{mm}}$$

Calculation of the **final** value of **creep deformation**:

$$w_{cc,\infty}=w(t=\infty)-w_{el}(t_0=3\,\text{nap})=0{,}093-0{,}033=\boxed{0{,}060\,\mathrm{mm}}$$

Example for the use of the effective modulus of elasticity for concrete

Elastic (uncracked) deformation of the RC cantilever beam **containig** also **effect of creep**:



$w_{el} = 0{,}033\ \text{mm}$

$w_{cc,\infty} = 0{,}060\ \text{mm}$

$w = 0{,}093\ \text{mm}$

Determination of the age of loading – t_0 – MSZ EN 4798-1:2004

Type of cement	Compressive strength grade of concrete			
	C8/10	**C12/15**	**C16/20**	**C20/25**
	Earliest time to remove the **side formwork,** [day]			
CEM 32,5	**3**	**2**	**2**	**1**
CEM 42,5	**-**	**2**	**1**	**1**
CEM 52,5	**-**	**-**	**1**	**1**

When assigning a shorter time than the deadlines given in **MSZ 4798-1: 2004 Table NAD L1**, it must be verified by a **strength test** that the compressive strength of the concrete is at least **3 N/mm^2**

[MSZ EN 4798-1:2004 – Section NAD L9 Formworking – Table NAD L1 – Page 138.]

Determination of the age of loading – t_0 – MSZ EN 4798-1:2004

Type of cement	Compressive strength grade of concrete					
	C16/20	**C20/25**	**C25/30**	**C30/37**	**C35/45**	**C40/50**
	Earliest time to remove of **load-bearing formwork** and **scaffolding**, [day]					
CEM 32,5	**21**	**20**	**19**	**17**	**15**	**-**
CEM 42,5	**18**	**17**	**15**	**12**	**10**	**10**
CEM 52,5	**14**	**13**	**12**	**10**	**8**	**6**

The dates given in **MSZ 4798-1: 2004 Table NAD L2** may be shortened if the strength test has confirmed that the concrete has previously reached **80% of the compressive strength** required at the age of **28 days**.

[MSZ EN 4798-1:2004 – Section NAD L9 Formworking – Table NAD L2 – Page 139.]

Determination of the age of loading – t_0 – Important notes

- **Removal** of **formwork** and *scaffolding* should be started when the **concrete strength is appropriate.**
- **During formworking**, the structure **must not be subjected** to **shocking**, **shaking** or **impact effects**.
- If there is a **phenomenon** or **defect** that **threatens the stability** during formwork, demolition **must be stopped** immediately.
- The formwork time – values given in **MSZ 4798-1:2004 Table NAD L1 and Table L2** – must be **extended by the number of days** while the average temperature was **below 0^0C**.
- The formwork times given in **MSZ 4798-1:2004 Table NAD L1 and Table L2** must be extended **by the designer** if the safety of the structure requires it (eg: high shrinkage deformation, thermal protection of the concrete).

[MSZ EN 4798-1:2004 – Section NAD L9 Formworking – Page 138.]

Non-linear notional creep of structural concrete – MSZ EN 1992-1-1:2010

When the compressive stress of concrete at an age t_0 exceeds the value $\mathbf{0{,}45 \times f_{ck}(t_0)}$ then **creep non-linearity should be considered**. Such a high stress can occur as a result of pretensioning, e.g in preceast concrete members at tendon level.

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (4) – Page 30.]

In such cases the **non-linear notional creep coefficient** should be obtained as follows:

$$\varphi_{nl}(\infty,t_0)=\varphi(\infty,t_0)\cdot e^{1{,}5\cdot\left(\frac{\sigma_c}{f_{cm}(t_0)}-0{,}45\right)}$$

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (4) – Page 30.]

Shrinkage of structural concrete – *fib* Bulletin 51

$$\varepsilon_c(t) = \varepsilon_c(t_0) + \varepsilon_{cc}(t,t_0) + \underbrace{\varepsilon_{cs}(t) + \varepsilon_{cT}(t,T)}_{\text{total stress independent concrete strain at a concrete age } t}$$

$\varepsilon_c(t)$ ➡ is the **total concrete strain** at age ***t***
$\varepsilon_c(t_0)$ ➡ is the stress-dependent **initial elastic strain** of concrete at time of stress application t_0
$\varepsilon_{cc}(t,t_0)$ ➡ is the **creep strain** of concrete at a concrete age $t \geq t_0$
$\varepsilon_{cs}(t)$ ➡ is the **shrinkage strain** of concrete at a concrete age ***t***
$\varepsilon_{cT}(t,T)$ ➡ is the **thermal strain** of concrete at a concrete age ***t***

[fib Bulletin 51 – Structural Concrete – Volume 1. – Section 3.1.6 – Page 53.-72.]

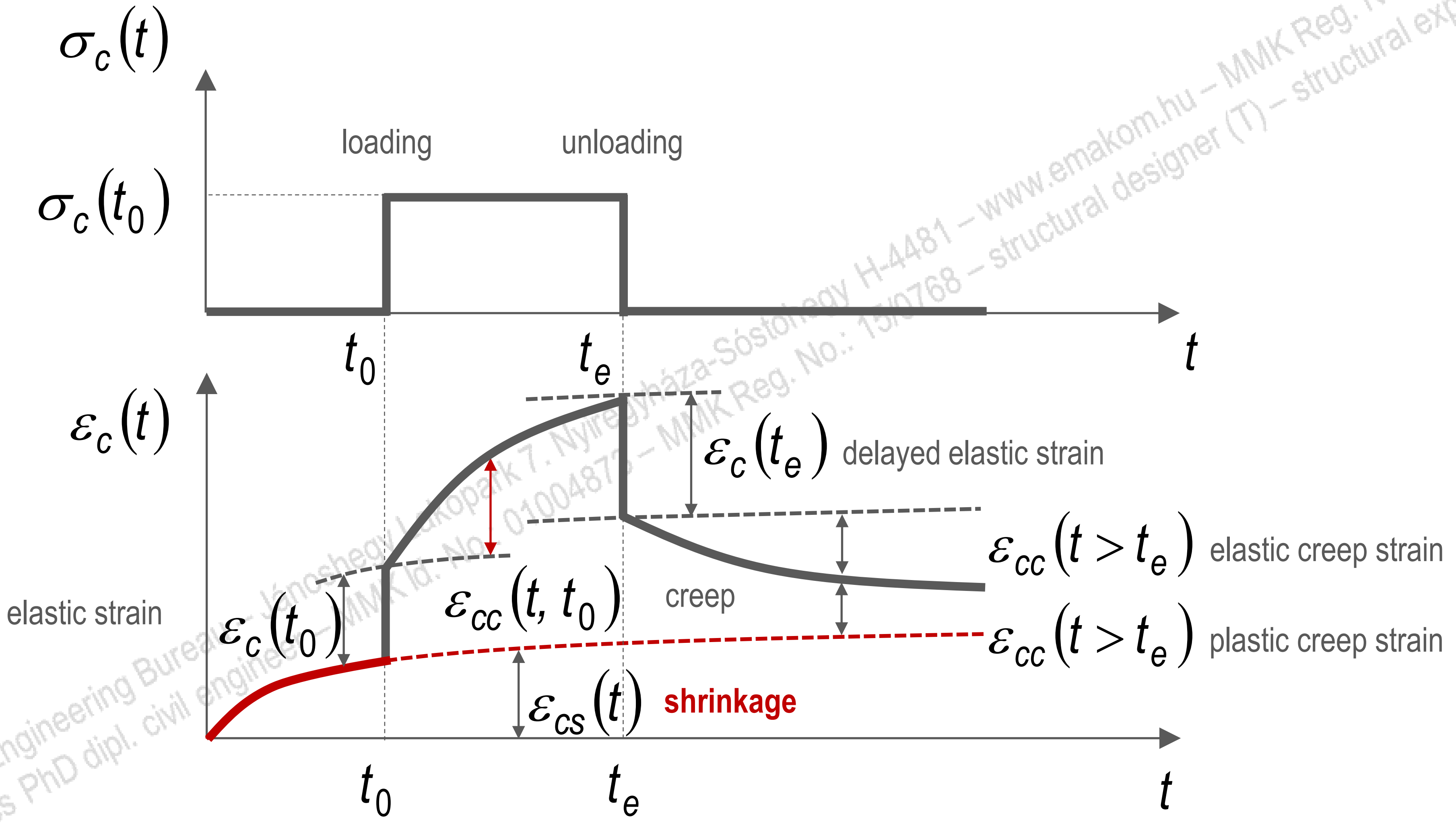
Shrinkage of structural concrete – MSZ EN 1992-1-1:2010

Shrinkage of the concrete depend on the:

- **environmental** conditions
  (environmental effects)
    - ambient **temperature**
    - ambient **humidity**
- **dimensions** of the element
  (size effect)
- **composition of concrete**
  (properties of constituents)
    - water to cement ratio
    - cement content, fineness of cement
    - fine aggregate content, fineness of fine aggregates
- **curing** of concrete
- degree of **hydration**
- **age of the concrete** at the time of the **onset of drying**

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (1)P – Page 30.]
[Tibor KAUSAY PhD: Creep and cracking of concrete http://www.betonopus.hu/notesz/kutyanyelv/zsugorodas.pdf]

Shrinkage of structural concrete – *fib* Bulletin 51



[fib Bulletin 51 – Structural Concrete – Volume 1. – Fig. 3.1-15 – Page 57.]

Types of shrinkage of structural concrete

Early or capillary or plastic shrinkage:

It occurs in the **fresh**, **plastic state of the concrete** and **creates capillaries**. The fresh concrete usually swells after short time of concreting, but when the water film disappears from its surface, it begins to shrink. **Early shrinkage is a rapid, short process** at the end of the curing time (usually **no more than 8 hours**. The resulting cracks are **hairline cracks on the surface**, their **width** can reach **1 - 2 mm**, in case of unfavorable climatic conditions their **total** value can be **4 mm/m**, but their **depth is small**, limited to the drying surface zone.

Chemical shrinkage:

It occurs because the **volume of hydration products** (cement stone) **is smaller than the volume of the initial cement paste** (cement + mixing water). Thus, during hydration, a decrease in volume occurs because water molecules are incorporated into the hydration product, i.e., the cement chemically binds a portion of the mixing water (approximately 25% by weight of the cement content). The other part of the water forms pore water (it is in a liquid state) and the third part evaporates.

Carbonation shrinkage:

Such process takes years. During the carbonation of concrete **minerals** (calcium carbonate) **are formed**, **their volumes are smaller than that of non-carbonated concrete**. During carbonation the previously bound water is also released, their evaporation increases the drying shrinkage. Thus, carbonation shrinkage of concrete is the result of a chemical and physical process.

[Tibor KAUSAY PhD: Creep and cracking of concrete, http://www.betonopus.hu/notesz/kutyanyelv/zsugorodas.pdf]

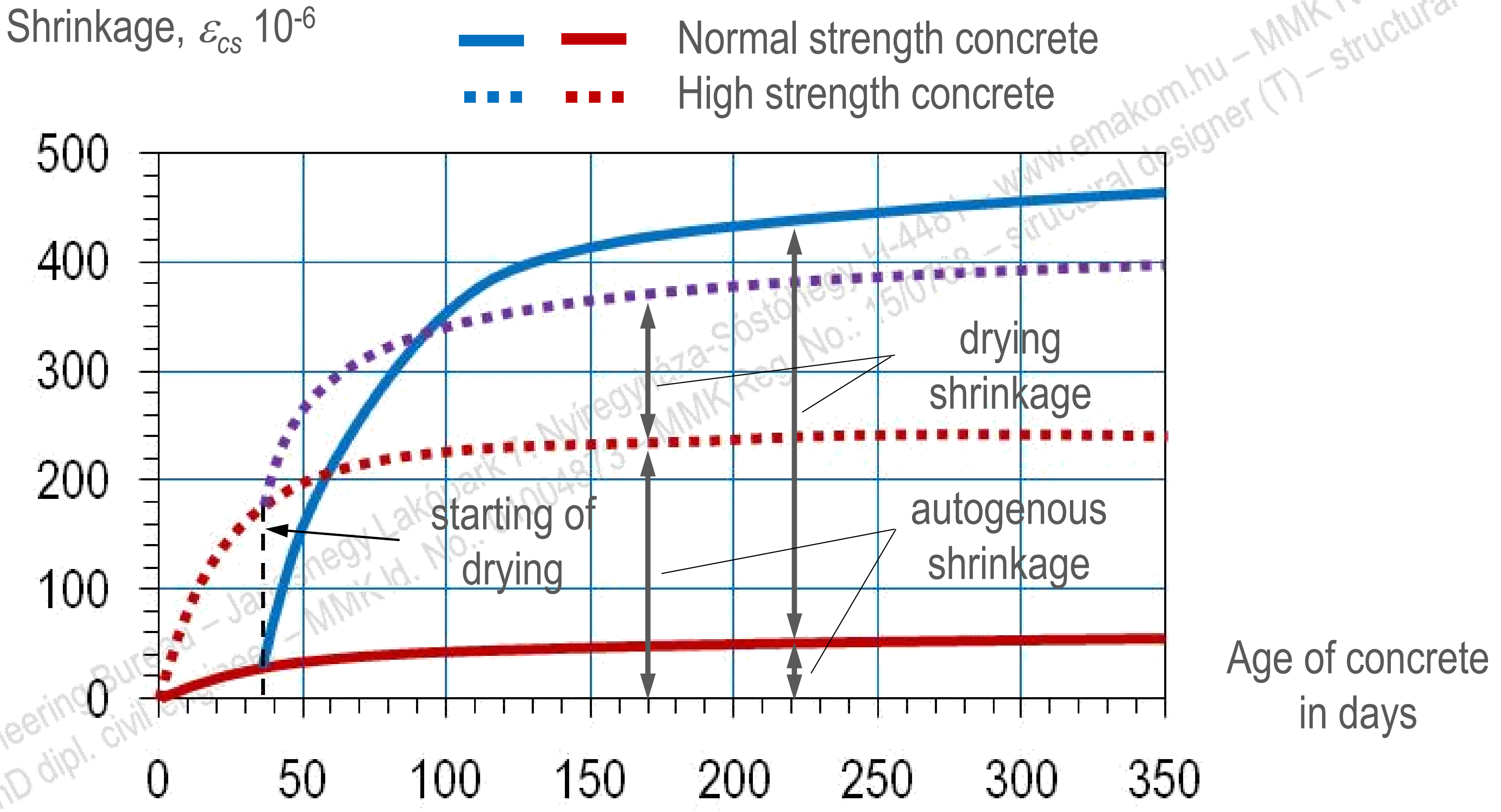
Types of shrinkage of structural concrete

Drying shrinkage:
It starts at the **end of the bond**, at the **beginning of hardening**, and **can take up to a year**, so it is a **slow process** that goes to its final value. Meanwhile, the pores of the cement stone dry out, the completely dried concrete does not shrink any further.

Autogenous shrinkage:
We are talking about autogenous **shrinkage in the case of concretes with a low water-cement ratio** (**w / c ≤ 0,4**) (these are usually high-strength, high-performance concretes). Its magnitude can be very significant, especially around **w / c = 0,3** and with a significant fines content (e.g. silica powder). Autogenic shrinkage begins at the time of early shrinkage and extends into the period of drying shrinkage. During hydration, the cement in the young concrete removes so much water from the pore structure that the concrete quickly reaches a state of mass stability at 70-90% relative humidity. **Concrete with a low water-cement ratio is made with so little water that hydration results in strong internal drying, which results in significant shrinkage.** Autogenous shrinkage is independent of the geometry of the structural element, and with hydration or resp. it is linearly related to the strength of the concrete. Autogenous shrinkage of concretes with a water-cement ratio of **w / c = 0,5** or more is negligible compared to drying shrinkage**.**

[Tibor KAUSAY PhD: Creep and cracking of concrete, http://www.betonopus.hu/notesz/kutyanyelv/zsugorodas.pdf]

Shrinkage of structural concrete – *fib* Bulletin 51



fib Bulletin 51, Structural Concrete, Volume 1, Fig. 3.1-16.

Shrinkage of structural concrete – MSZ EN 1992-1-1:2010

The **total shrinkage strain is composed of two components**, the **drying shrinkage strain** and the **autogenous shrinkage strain**. The **drying shrinkage strain develops slowly**, since it is a function of the migration of the water through the hardenen concrete. The **autogenous shrinkage strain develops during hardening of the concrete**: the major part therefore develops in the **early days after casting**. **Autogeous shrinkage is a linear function of the concrete strength**. It should be considered specifically when new concrete is cast against hardened concrete. Hence tha values of the total shrinkage strain ε_{cs} follow from:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$$

ε_{cs} ➡ is the **total shrinkage** strain
ε_{cd} ➡ is the **drying shrinkage** strain
ε_{ca} ➡ is the **autogenous shrinkage** strain

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (6) – Page 32.]

Shrinkage of structural concrete – MSZ EN 1992-1-1:2010

The **final value of the drying shrinkage strain**, $\varepsilon_{cd,\infty}$ is equal to:

$$\varepsilon_{cd,\infty} = k_h \cdot \varepsilon_{cd,0}$$

h_0	k_h
100	1,00
200	0,85
300	0,75
≥ 500	0,70

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Table 3.3 – Page 33.]

$f_{ck}/f_{ck\ cube}$ [MPa]	$\varepsilon_{cd,0}$ [$^0/_{00}$] (Class N)					
	Relative humidity [%]					
	20	**40**	**60**	**80**	**90**	**100**
20/25	0,62	0,58	0,49	0,30	0,17	0,00
40/50	0,48	0,46	0,38	0,24	0,13	0,00
60/75	0,38	0,36	0,30	0,19	0,10	0,00
80/95	0,30	0,28	0,24	0,15	0,08	0,00
90/105	0,27	0,25	0,21	0,13	0,07	0,00

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Table 3.2 – Page 32.]

Development of drying shrinkage strain in time – MSZ EN 1992-1-1:2010

The development of **the drying shrinkage strain in time** follows from:

$$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0}$$

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s) + 0{,}04 \cdot \sqrt{{h_0}^3}}$$

t ➡ is the **age of concrete at the moment considered** in days

t_s ➡ is **the age of the concrete** (days) **at the beginning of drying shrinkage** (or swelling) Normally this is at the end of curing.

$h_0 = \dfrac{2 \cdot A_c}{u}\,[\mathrm{mm}]$ ➡ is the **notional size** (mm) of the cross-section

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (6) – Page 33.]

Autogenous shrinkage strain of structural concrete – MSZ EN 1992-1-1:2010

The **autogenous shrinkage strain** follows from:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

$$\varepsilon_{ca}(\infty) = 2{,}5 \cdot (f_{ck} - 10) \cdot 10^{-6}$$

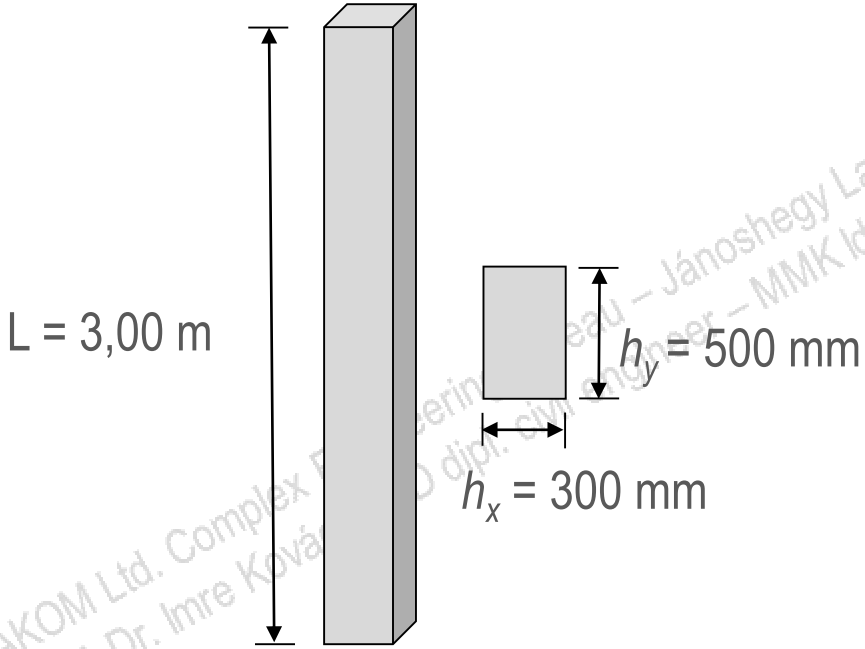
$$\beta_{as}(t) = 1 - e^{\left(-0{,}2 \cdot t^{0{,}5}\right)}$$

$\varepsilon_{ca}(t)$	➡	is the **autogenous shrinkage strain** at the concrete age ***t***
$\varepsilon_{ca}(\infty)$	➡	is the **final value of the autogenous shrinkage**
t	➡	is the **age of the concrete** at the moment **considered**

[MSZ EN 1992-1-1:2010 – Section 3.1.4 – Paragraph (6) – Page 33.]

Example for the determination of shrinkage strain and deformation

Determine the final shrinkage deformation of a reinforced concrete column using the following cross-section: **300/500 mm**, height**: L = 3000 mm**, grade of concrete: **C20/25**, type of aggregate: **sandy gravel**, type of cement: **CEM I 52.5 R** under inside conditions (**RH = 50%**) and a **value of 7 days** if the **curing of concrete ends 2 days after concreting** .



Environmental conditions:	**Relative humidity 50%**
End of curing:	t_0 **= 2 days**
Moments considered in calculation:	**7 days, final value**
Type of cement:	**CEM I. 52,5 R** (**Class R**)
Concrete strength grade:	**C20/25**
Type of aggragate:	**sandy gravel**

Example for the determination of shrinkage strain and deformation

Calculation of the **notional size**:

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 300 \cdot 500}{2 \cdot (300 + 500)} = \boxed{187{,}5\ \text{mm}}$$

Determination of the **final value of the drying shrinkage strain** of concrete **(*t* = ∞)**:

$$\varepsilon_{cd,\infty} = k_h \cdot \varepsilon_{cd,0} = 0{,}87 \cdot 0{,}54 = \boxed{0{,}470\ ‰}$$

$$k_h\left(h_0 = 187{,}5\ \text{mm}\right) = 0{,}87$$

Determination of the **final value of the drying shrinkage deformation (*t* = ∞)**:

$$\varepsilon_{cd,0}\left(C20/25, \text{RH} = 50\%\right) = 0{,}54\ ‰$$

$$\Delta L_{cd,\infty} = \varepsilon_{cd,\infty} \cdot L = 0{,}470\ ‰ \cdot 3000 = \boxed{1{,}41\ \text{mm}}$$

Determination of the **drying shrinkage strain** at the concrete **age 7 days (*t* = 7 days):**

$$\varepsilon_{cd,7} = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0} = \frac{(t - t_s)}{(t - t_s) + 0{,}04 \cdot \sqrt{h_0^3}} \cdot k_h \cdot \varepsilon_{cd,0} = \frac{(7 - 2)}{(7 - 2) + 0{,}04 \cdot \sqrt{187{,}5^3}} \cdot 0{,}87 \cdot 0{,}54 = \boxed{0{,}022\ ‰}$$

Determination of the **drying shrinkage deformation** at the concrete **age 7 days**:

$$\Delta L_{cd,7} = \varepsilon_{cd,7} \cdot L = 0{,}022\ ‰ \cdot 3000 = \boxed{0{,}066\ \text{mm}}$$

Example for the determination of shrinkage strain and deformation

Determination of the **final value of the autogenous shrinkage strain** of concrete **(*t* = ∞):**

$$\varepsilon_{ca}(\infty)=2{,}5\cdot\left(f_{ck}-10\right)\cdot10^{-6}=2{,}5\cdot\left(20-10\right)\cdot10^{-6}=\boxed{0{,}025\,{}^{0}\!/_{00}}$$

Determination of the **final value of the autogenous shrinkage deformation (*t* = ∞):**

$$\Delta L_{ca,\infty}=\varepsilon_{ca,\infty}\cdot L=0{,}025\,{}^{0}\!/_{00}\cdot3000=\boxed{0{,}075\,\mathrm{mm}}$$

Determination of the **autogenous shrinkage strain** at the concrete age 7 days **(*t* = 7 days):**

$$\varepsilon_{ca}(t)=\beta_{as}(t)\cdot\varepsilon_{ca}(\infty)=\left[1-e^{\left(-0{,}2\cdot t^{0{,}5}\right)}\right]\cdot\varepsilon_{ca}(\infty)=\left[1-e^{\left(-0{,}2\cdot7^{0{,}5}\right)}\right]\cdot0{,}025\,{}^{0}\!/_{00}=\boxed{0{,}009\,{}^{0}\!/_{00}}$$

Determination of the **autogenous shrinkage deformation** at the concrete age 7 days **(*t* = 7 days):**

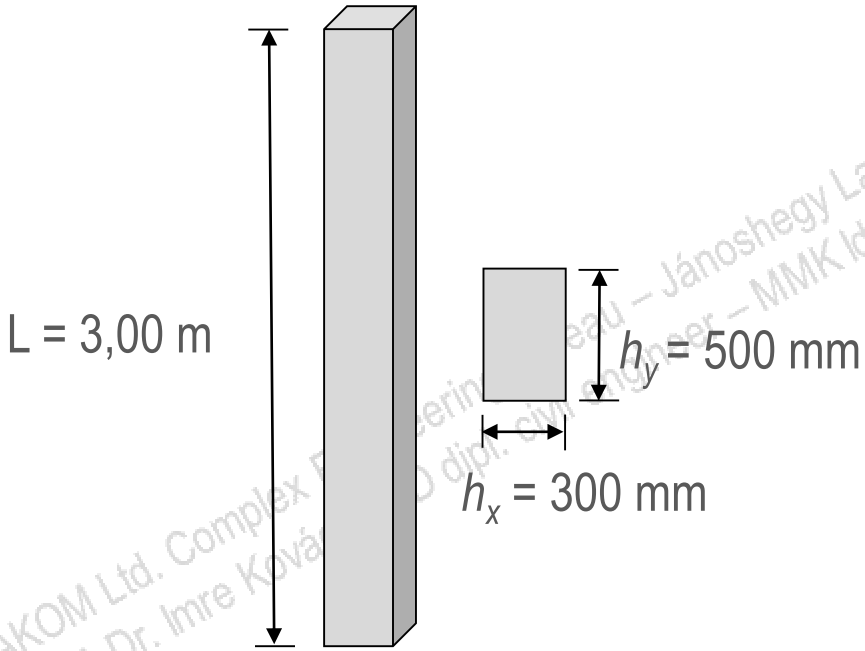
$$\Delta L_{ca,7}=\varepsilon_{ca,7}\cdot L=0{,}009\,{}^{0}\!/_{00}\cdot3000=\boxed{0{,}027\,\mathrm{mm}}$$

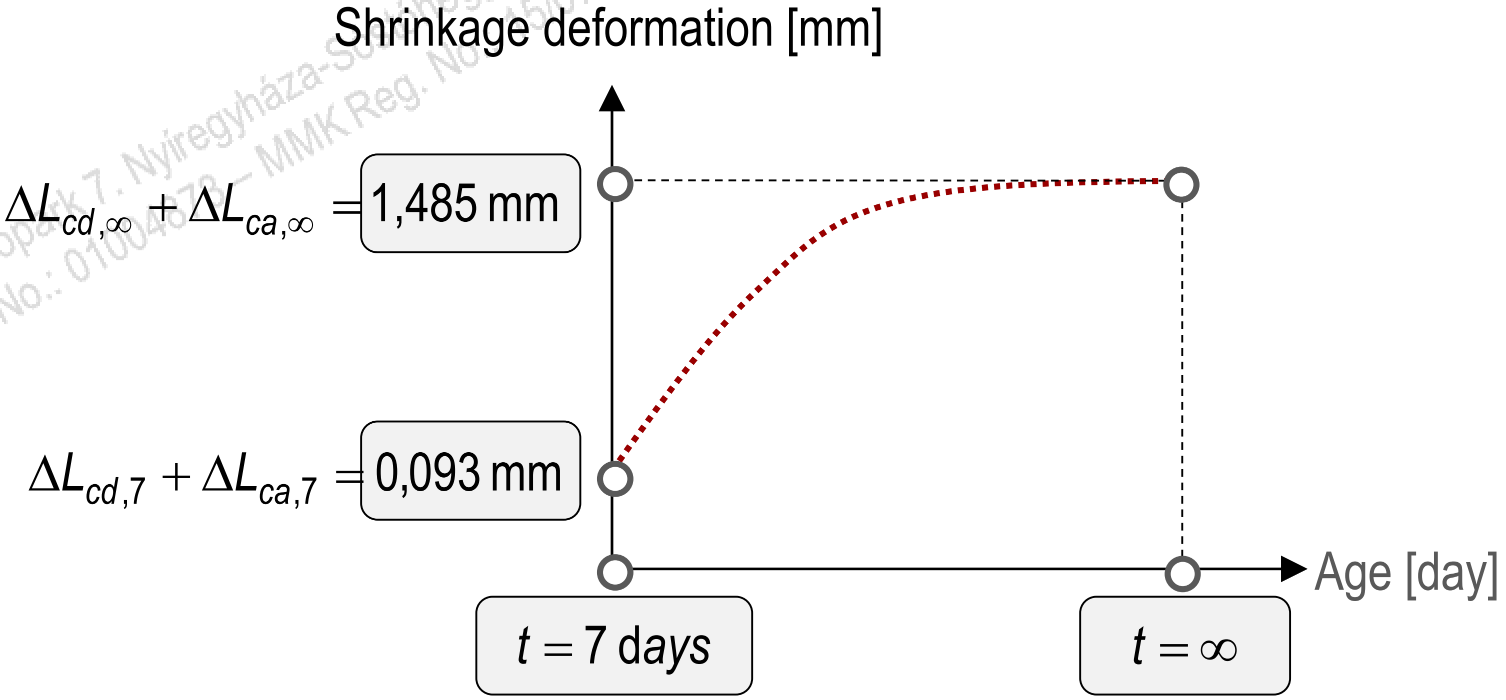
Example for the determination of shrinkage strain and deformation

About **5%** of the **total drying shrinkage** (**1,410 mm**) **takes place** in the **first 7 days** (**0,066 mm**)!!!

About **35%** of the **autogenous shrinkage** (**0,075 mm**) **takes place** in the **first 7 days** (**0,027 mm**)!!!

About **6%** of **the total shrinkage** (**1,485 mm**) **takes place** in the **first 7 days** (**0,093 mm**)!!!





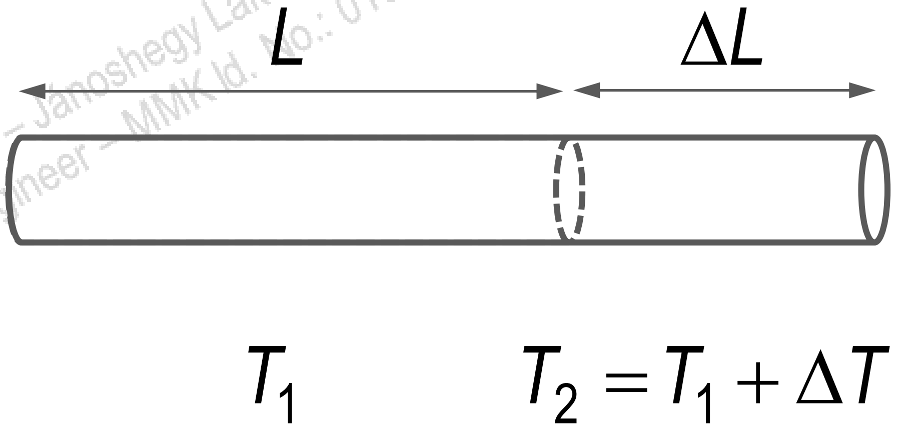
Linear coefficient of thermal expansion – MSZ EN 1992-1-1:2010

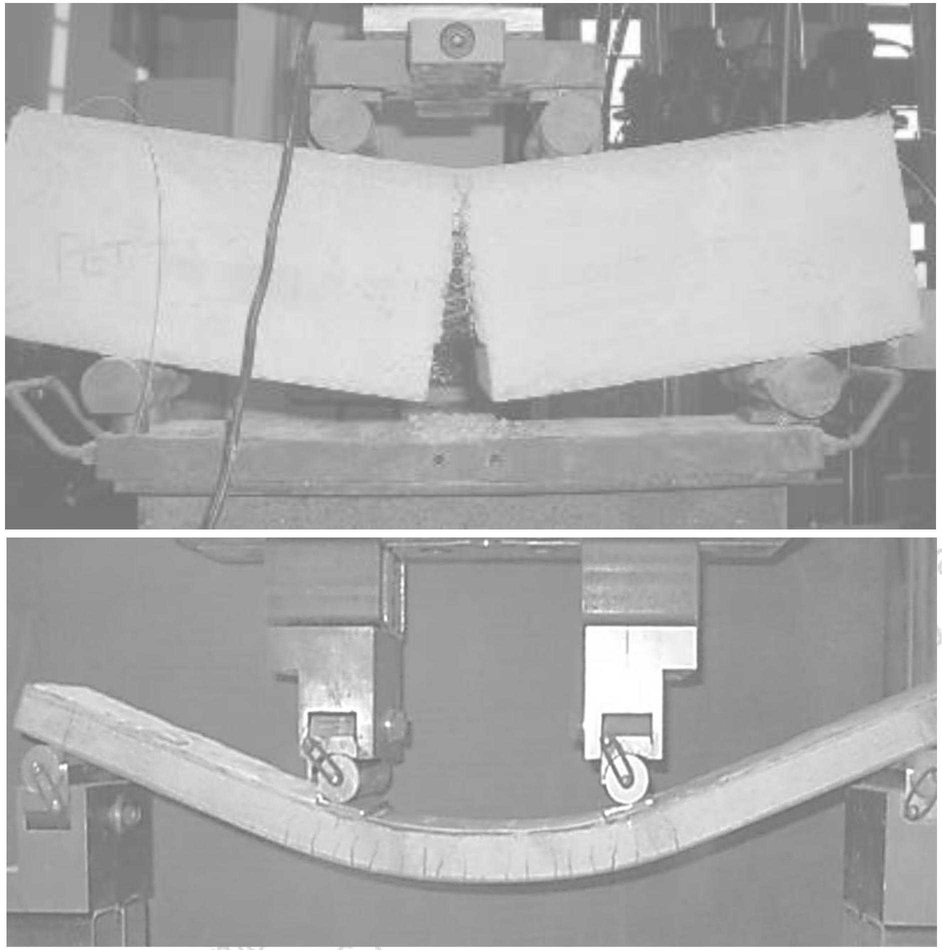
Unless more accurate information is available, the **linear coefficient of thermal expansion** may be taken equal to **10×10^{-6} 1/◦K**.

[MSZ EN 1992-1-1:2010 – Section 3.1.3 – Paragraph (5) – Page 30.]

Linear coefficient of thermal expansion:

$$\alpha = \frac{\Delta L}{L}\cdot\frac{1}{\Delta T}$$





Reinforced Concrete (RC) Structures

Topic 5.

Strain properties of structural concrete

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Thank you for your kind attention!