Reinforced Concrete (RC) Structures

Topic 14.

Cracked state of stress - II. state of stress

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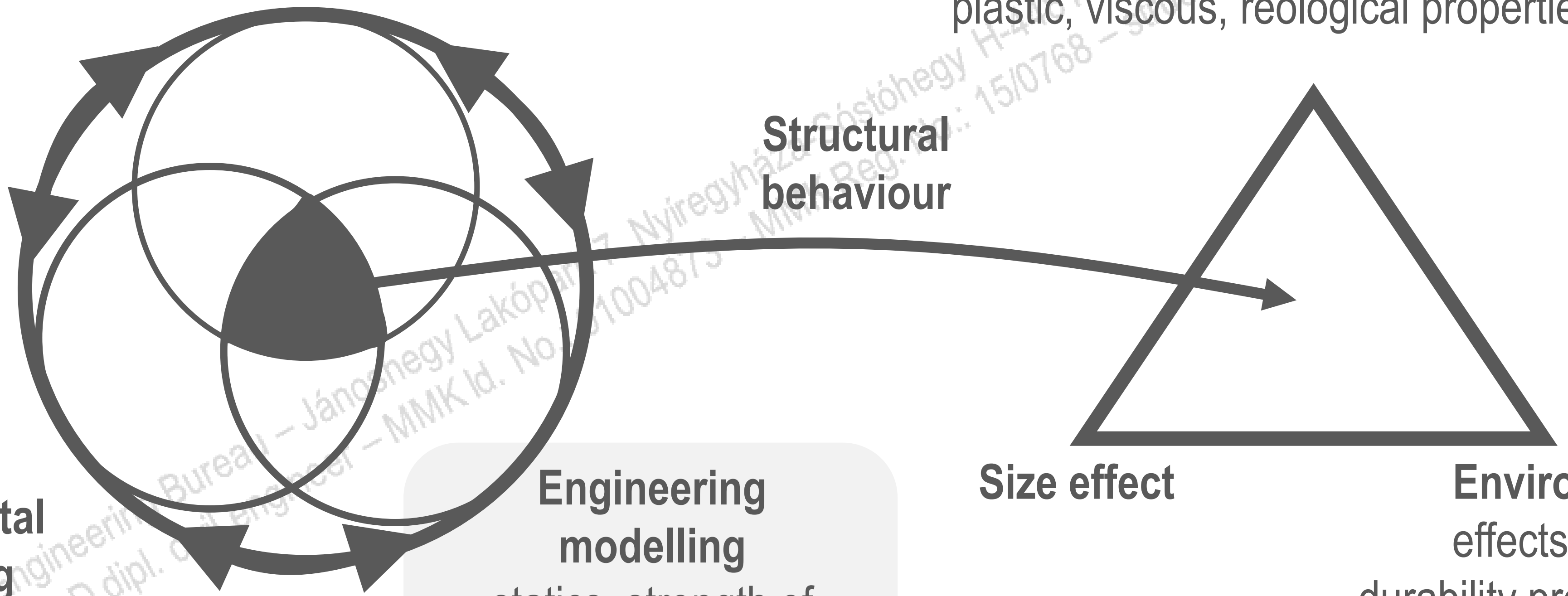
Modeling of structural behaviour of RC members

Numerical modelling
linear, non linear, plastic, 1st order theory, 2nd order theory, etc.

Material properties
homogen, inhomogen, izotropic, anizotropic
elastic linear, non linear elastic,
plastic, viscous, reological properties, etc.



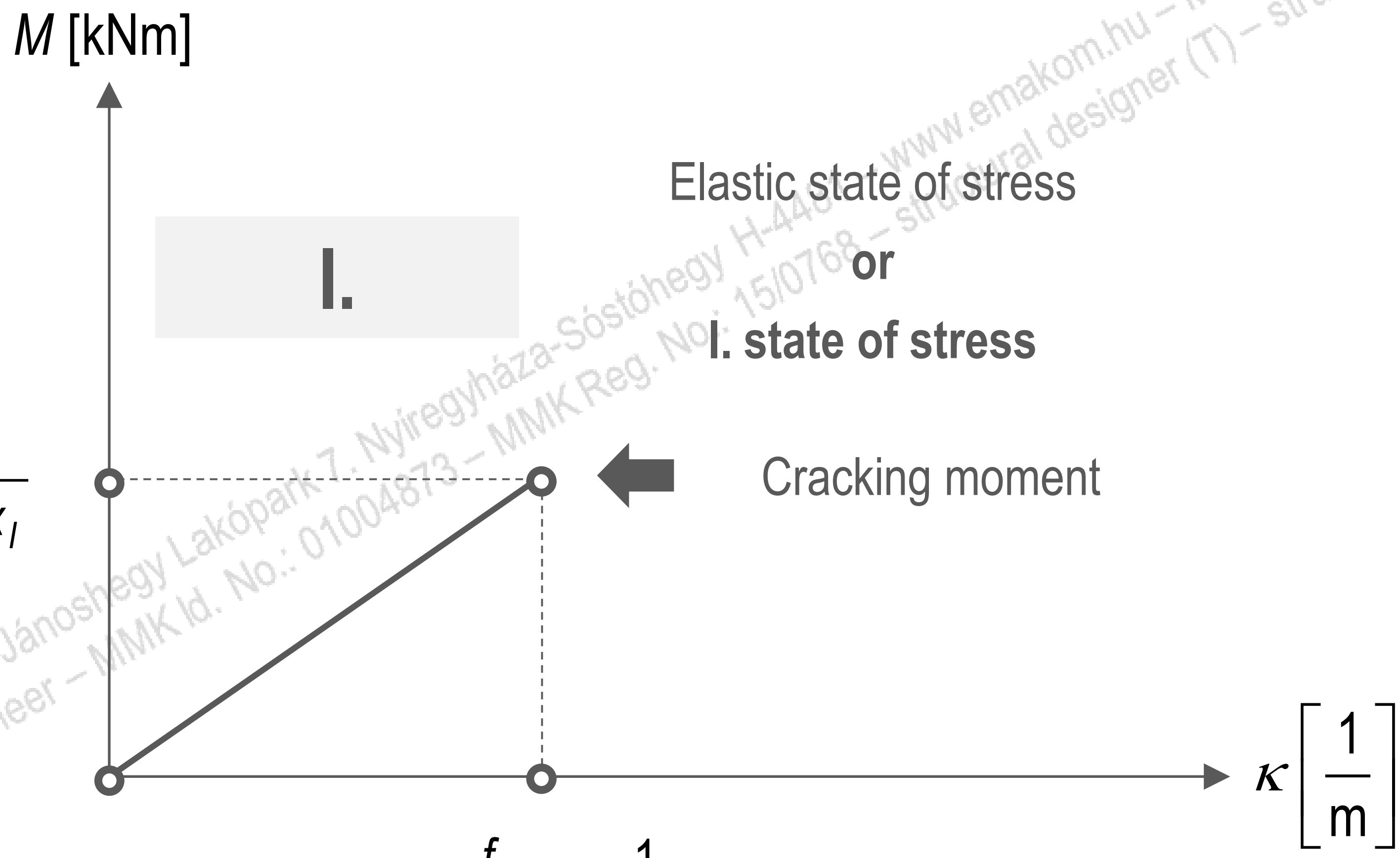
Experimental modelling
full-scale,
non full-scale, etc.

Engineering modelling
statics, strength of materials, elasticity, plasticity, dynamics, etc.

Size effect

Environment
effects, loads,
durability properties, etc.

RC cross-section in the end-point of the uncracked state – Cracking



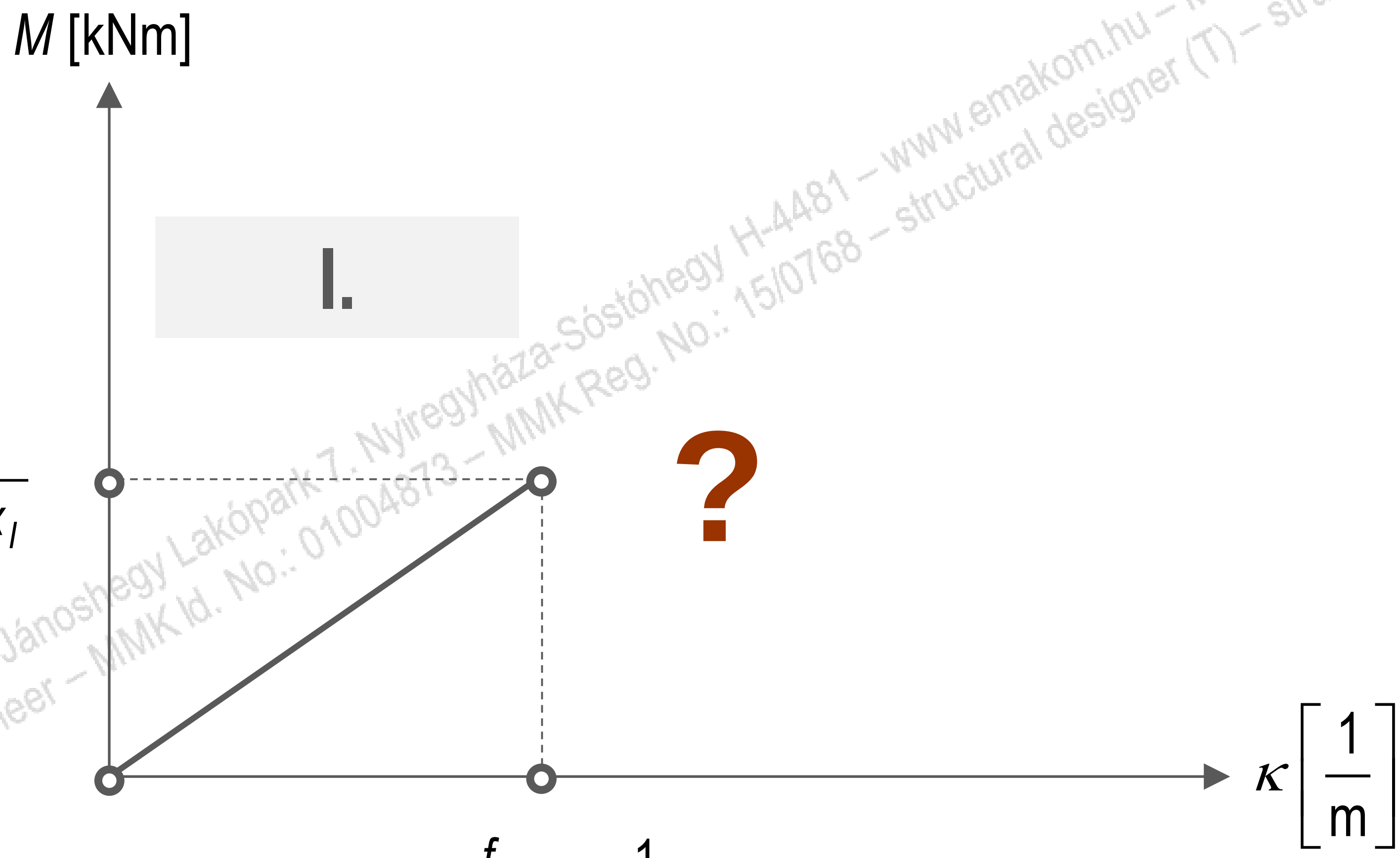
Elastic state of stress
or
I. state of stress

Cracking moment

$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I}$$

$$\kappa_I = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h - x_I}$$

RC cross-section after the end-point of the uncracked state – Cracked state



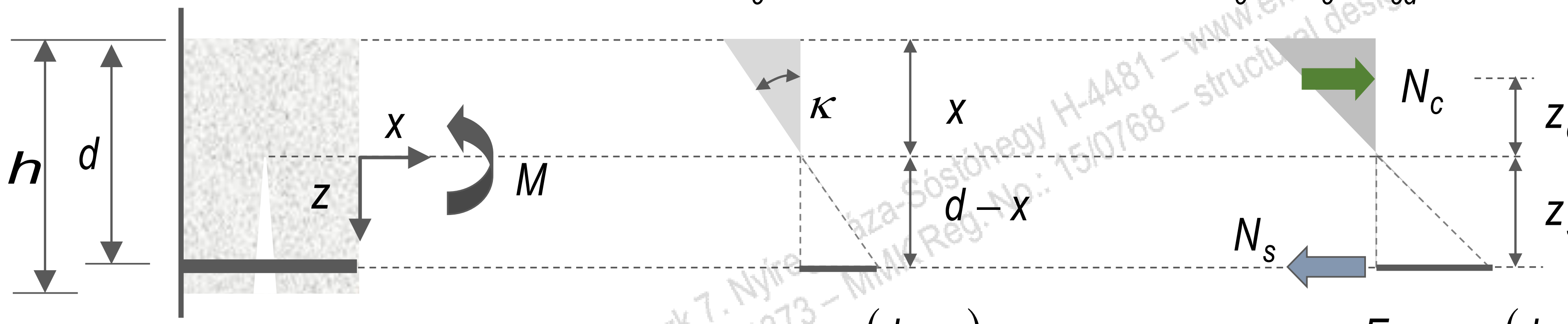
$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I}$$

$$\kappa_I = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h - x_I}$$

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_s = \kappa \cdot (d - x) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$$

1. Horizontal force equilibrium : Internal forces: $N_c = \frac{1}{2} \cdot (\kappa \cdot x) \cdot E_{cd} \cdot x \cdot b$ Arm of the internal forces: $z_c = \frac{2}{3} \cdot x$

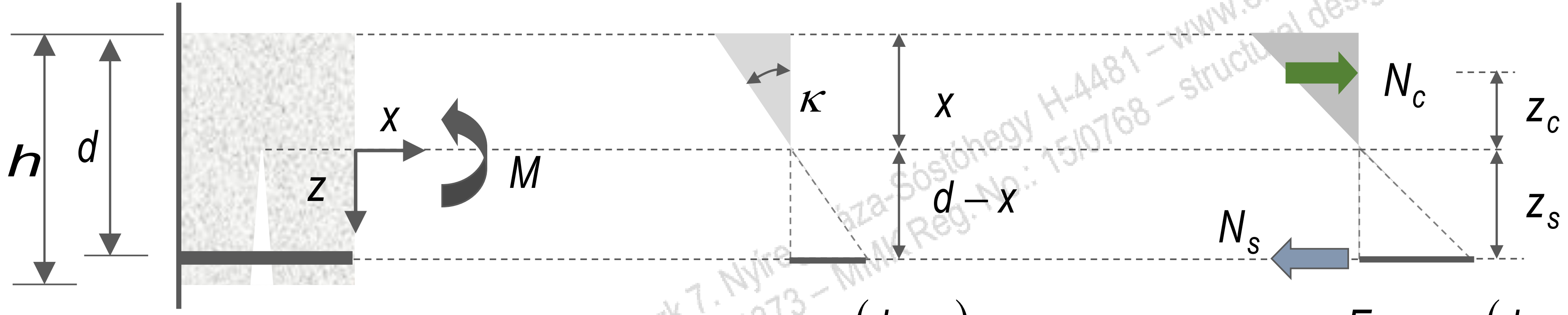
$\Sigma N = 0 \rightarrow 0 = N_c - N_s$ $N_s = \kappa \cdot (d - x) \cdot E_s \cdot A_s$ $z_s = (d - x)$

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$\varepsilon_c = \kappa \cdot x$

$\sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$



$\varepsilon_s = \kappa \cdot (d - x)$

$\sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$

1. Horizontal force equilibrium:

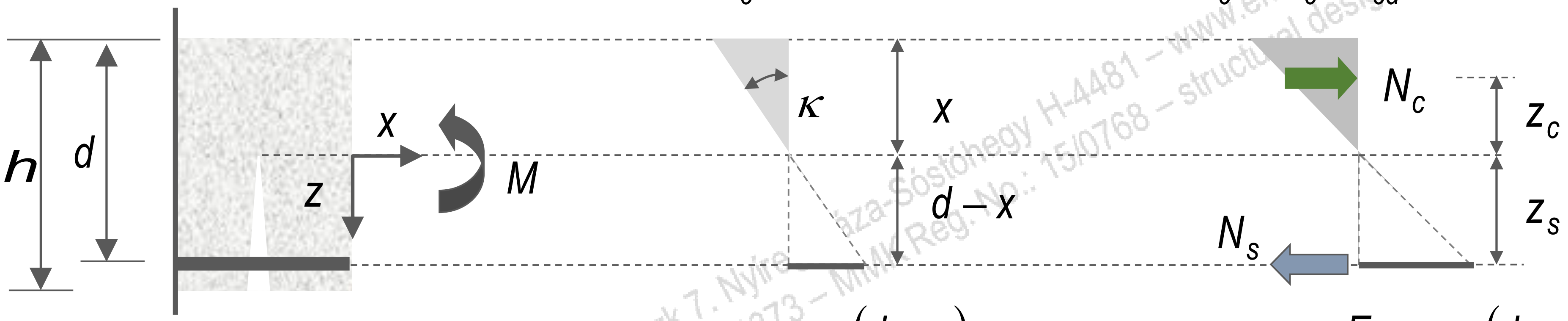
$$\Sigma N = 0 \quad \rightarrow \quad 0 = N_c - N_s$$

$$0 = \frac{1}{2} \cdot (\kappa \cdot x) \cdot E_{cd} \cdot x \cdot b - \kappa \cdot (d - x) \cdot E_s \cdot A_s$$

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_s = \kappa \cdot (d - x) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$$

1. Horizontal force equilibrium:

$$\Sigma N = 0 \quad \rightarrow \quad 0 = N_c - N_s$$

Static moment on the axis of bending

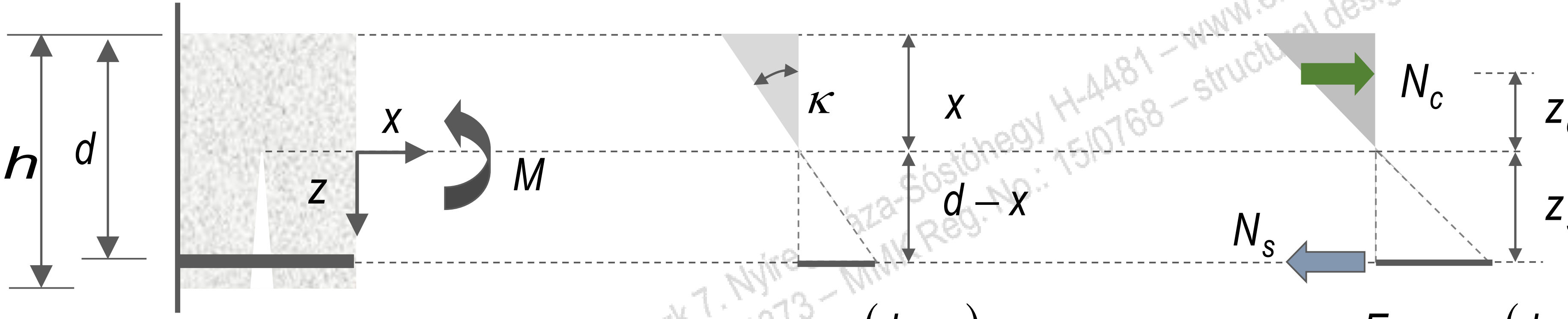
$$0 = \kappa \cdot \left\{ \frac{1}{2} \cdot b \cdot x^2 - (d - x) \cdot \frac{E_s}{E_{cd}} \cdot A_s \right\} \quad \Rightarrow \quad \alpha = \frac{E_s}{E_{cd}} \quad \Rightarrow \quad 0 = \frac{1}{2} \cdot b \cdot x^2 - (d - x) \cdot \alpha \cdot A_s$$

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_c = \kappa \cdot x$$

$$\sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_s = \kappa \cdot (d - x)$$

$$\sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$$

1. Horizontal force equilibrium :

$$\Sigma N = 0 \quad \rightarrow \quad 0 = N_c - N_s$$

Neutral axis / Depth of the compressed belt

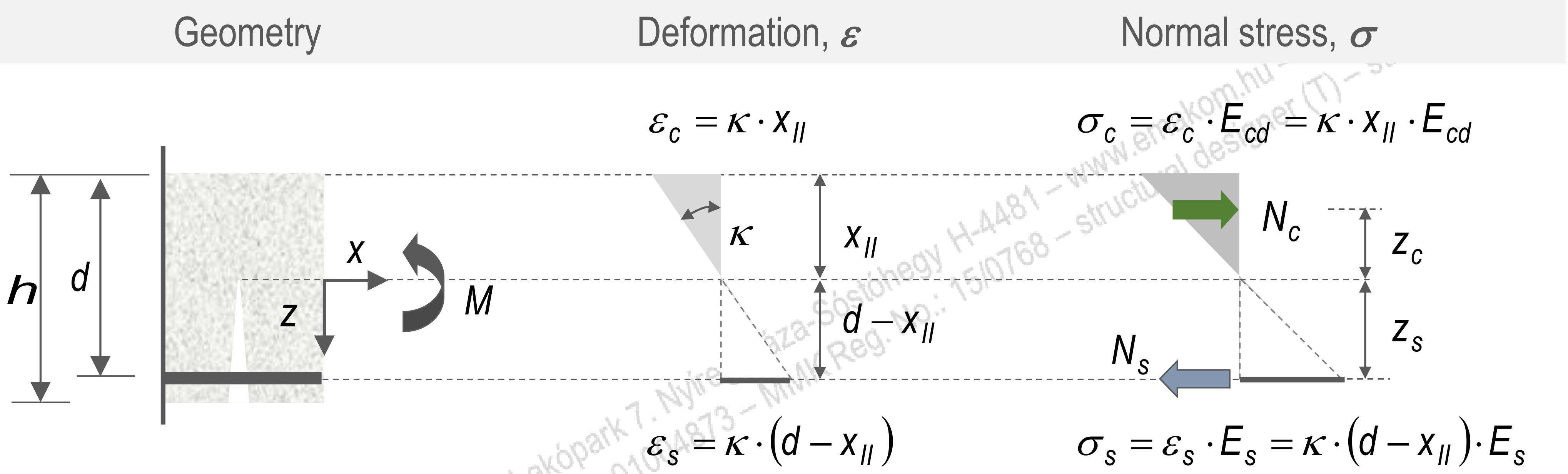
$$0 = \frac{1}{2} \cdot b \cdot x^2 - (d - x) \cdot \alpha \cdot A_s$$

$$a \cdot x^2 + b \cdot x + c = 0$$

$$b \cdot x^2 + (2 \cdot \alpha \cdot A_s) \cdot x - 2 \cdot \alpha \cdot A_s \cdot d = 0$$

$$x = x_{II} = -\frac{\alpha \cdot A_s}{b} + \sqrt{\left(\frac{\alpha \cdot A_s}{b}\right)^2 + \frac{\alpha \cdot A_s}{b} \cdot 2 \cdot d}$$

Load process of reinforced concrete member in bending – elastic state / cracked state



2. Bending moment equilibrium: $\Sigma M = 0 \quad \rightarrow \quad M = N_c \cdot z_c + N_s \cdot z_s$

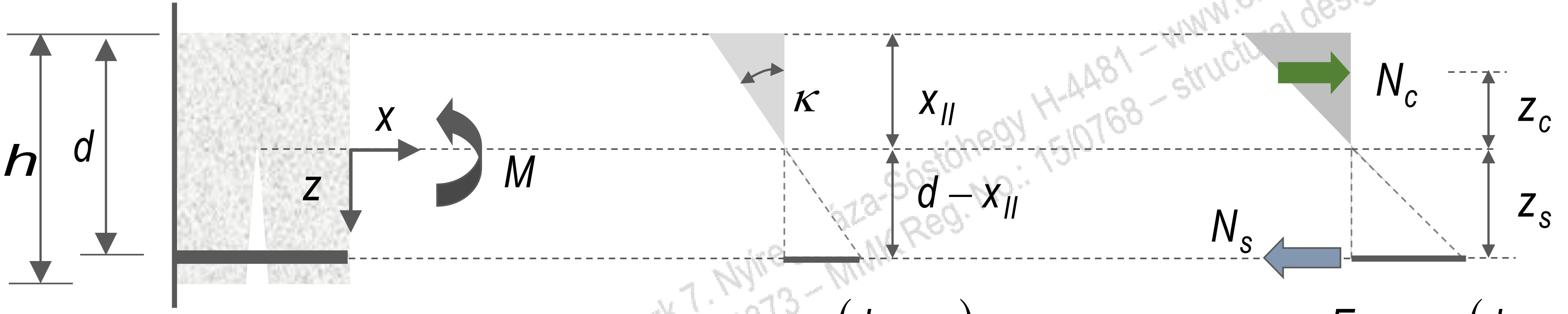
$$M = \left\{ \frac{1}{2} \cdot (\kappa \cdot x_{II}) \cdot E_{cd} \cdot b \cdot x_{II} \right\} \cdot \frac{2}{3} \cdot x_{II} + \kappa \cdot (d - x_{II}) \cdot E_s \cdot A_s \cdot (d - x_{II})$$

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_c = \kappa \cdot x_{II}$$

$$\sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x_{II} \cdot E_{cd}$$



$$\varepsilon_s = \kappa \cdot (d - x_{II})$$

$$\sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x_{II}) \cdot E_s$$

2. Bending moment equilibrium:

$$\Sigma M = 0 \quad \rightarrow \quad M = N_c \cdot z_c + N_s \cdot z_s$$

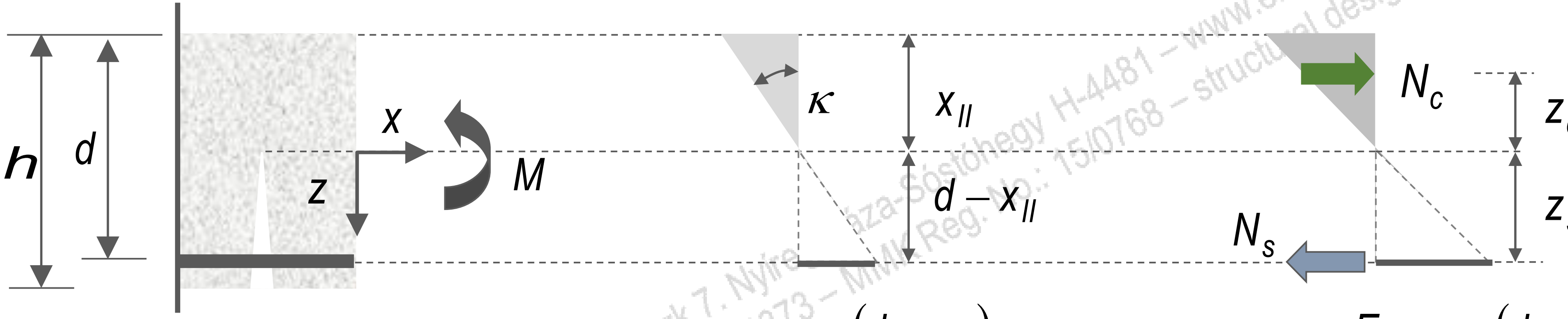
Moment of inertia of the elastic/cracked cross-section

$$M = \kappa \cdot E_{cd} \cdot \left\{ \frac{b \cdot x_{II}^{\,3}}{3} + \alpha \cdot A_s \cdot (d - x_{II})^2 \right\} \quad \Rightarrow \quad M = \kappa \cdot E_{cd} \cdot I_{II}$$

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_c = \kappa \cdot x_{II} \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x_{II} \cdot E_{cd}$$



$$\varepsilon_s = \kappa \cdot (d - x_{II}) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x_{II}) \cdot E_s$$

$$\kappa = \frac{\varepsilon_s}{d - x_{II}} = \frac{\sigma_s}{E_s} \cdot \frac{1}{d - x_{II}} \;\Rightarrow$$

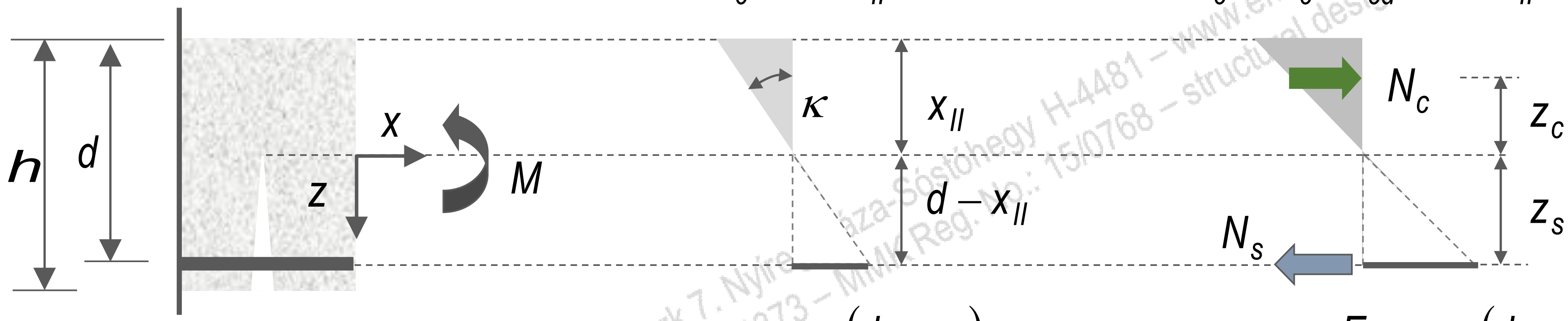
$$M = \kappa \cdot E_{cd} \cdot I_{II} = \frac{\sigma_s}{E_s} \cdot E_{cd} \cdot \frac{\left\{ \frac{b \cdot x_{II}^3}{3} + \alpha \cdot A_s \cdot (d - x_{II})^2 \right\}}{d - x_{II}} = \frac{\sigma_s}{\alpha} \cdot \frac{I_{II}}{d - x_{II}} = \frac{\sigma_s}{\alpha} \cdot W_{II}^s$$

Cross-sectional moduli of the cross-section on the tensioned reinforcement

Load process of reinforced concrete member in bending – elastic state / cracked state

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_c = \kappa \cdot x_{II} \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x_{II} \cdot E_{cd}$$

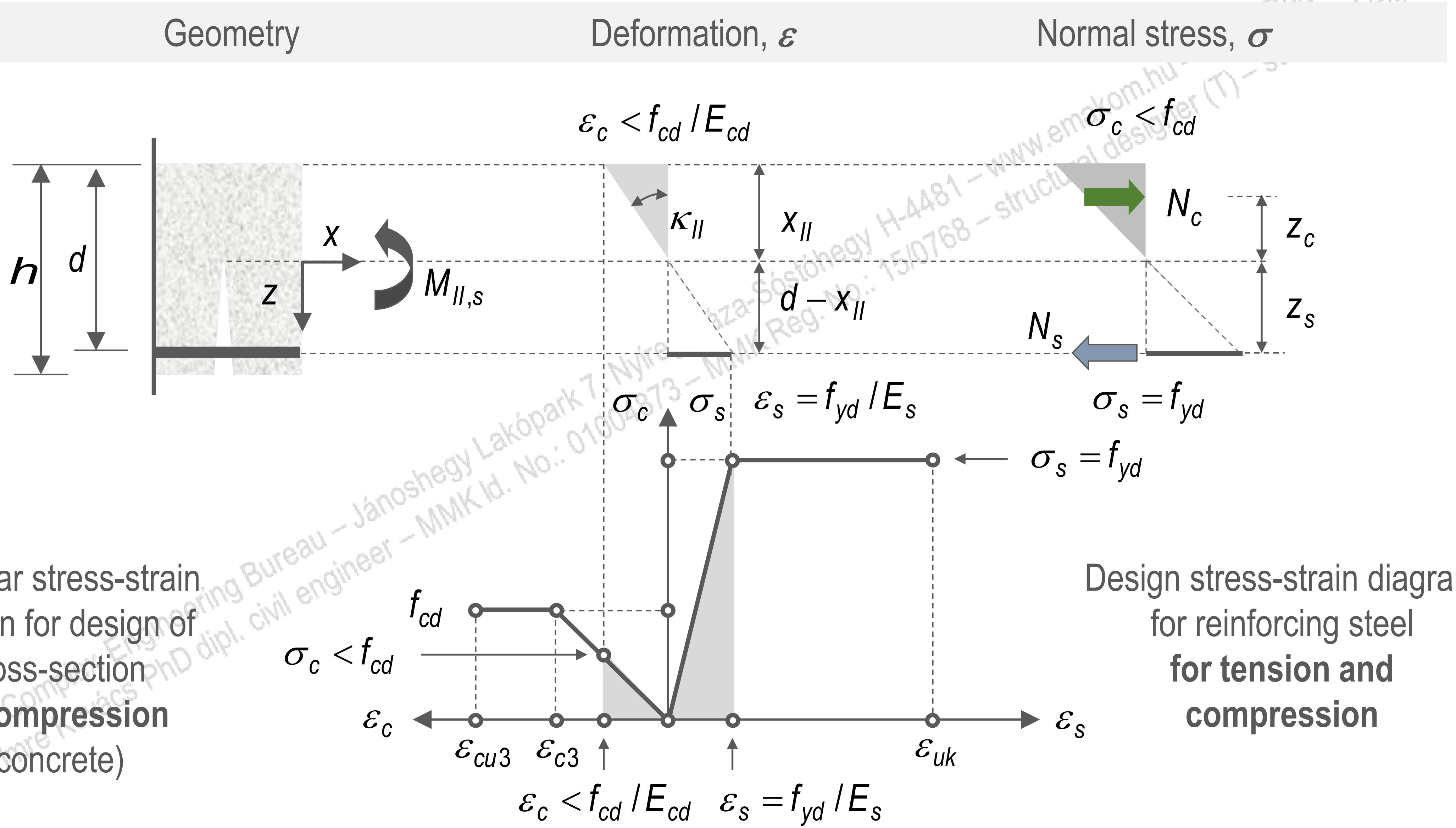


$$\varepsilon_s = \kappa \cdot (d - x_{II}) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x_{II}) \cdot E_s$$

Cross-sectional moduli of the cross-section on the most compressed fibre of the cross-section...

$$\kappa = \frac{\varepsilon_c}{x_{II}} = \frac{\sigma_c}{E_{cd}} \cdot \frac{1}{x_{II}} \Rightarrow M = \kappa \cdot E_{cd} \cdot I_{II} = \frac{\sigma_c}{E_{cd}} \cdot E_{cd} \cdot \frac{\left\{ \frac{b \cdot x_{II}^{3}}{3} + \alpha \cdot A_s \cdot (d - x_{II})^2 \right\}}{x_{II}} = \sigma_c \cdot \frac{I_{II}}{x_{II}} = \sigma_c \, W_{II}^{c}$$

The first plastic phenomenon is the yielding of the reinforcemen – $\sigma_s = f_{yd}$

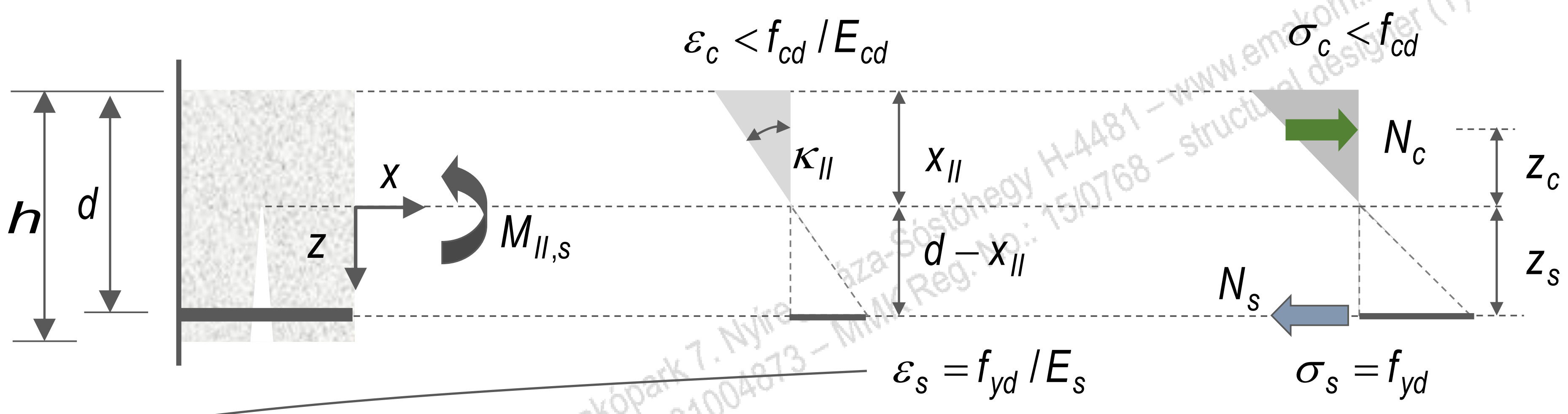


Bi-linear stress-strain relation for design of cross-section **in compression** (concrete)

Design stress-strain diagram for reinforcing steel **for tension and compression**

The first plastic phenomenon is the yielding of the reinforcemen – $\sigma_s = f_{yd}$

Geometry	Deformation, ε	Normal stress, σ

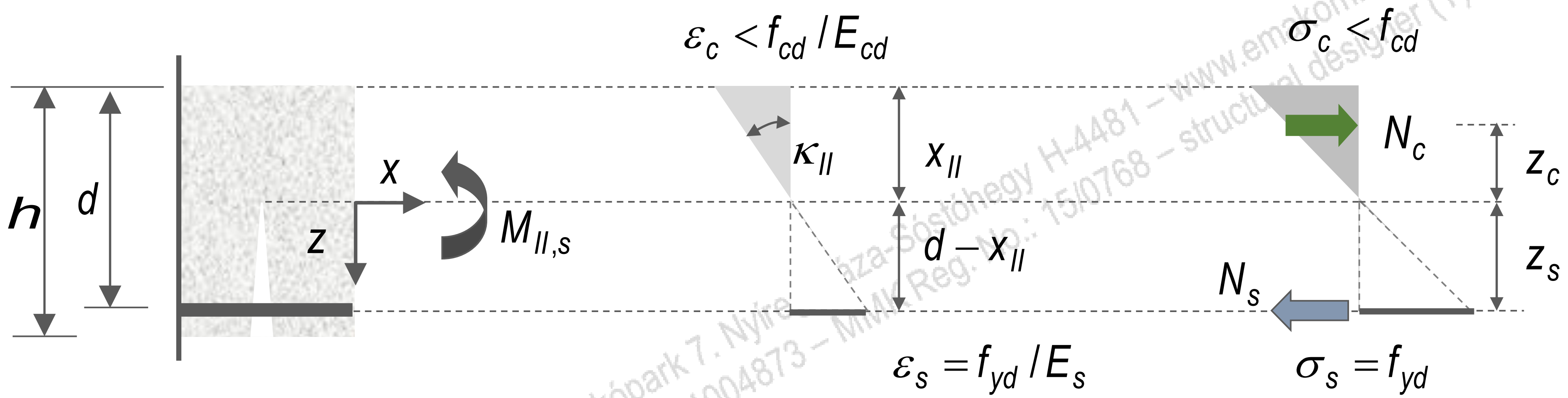


Bending moment corresponds to **yielding of reinforcement** – end of the elastic/cracked state of stress or end of the II. state of stress

$$\kappa_{II} = \frac{\varepsilon_s}{d - x_{II}} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}} \Rightarrow M_{II} = \kappa_{II} \cdot E_{cd} \cdot I_{II} = \frac{f_{yd}}{E_s} \cdot E_{cd} \cdot \frac{\left\{ \frac{b \cdot x_{II}^3}{3} + \alpha \cdot A_s \cdot (d - x_{II})^2 \right\}}{d - x_{II}} = \frac{f_{yd}}{\alpha} \cdot \frac{I_{II}}{d - x_{II}} = \frac{f_{yd}}{\alpha} \cdot W_{II}^s$$

The first plastic phenomenon is the yielding of the reinforcemen – $\sigma_s = f_{yd}$

Geometry	Deformation, ε	Normal stress, σ



Curveture corresponds to the yielding of reinforcement:

$$\kappa_{II} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}}$$

Bending moment corresponds to the yielding of reinforcement:

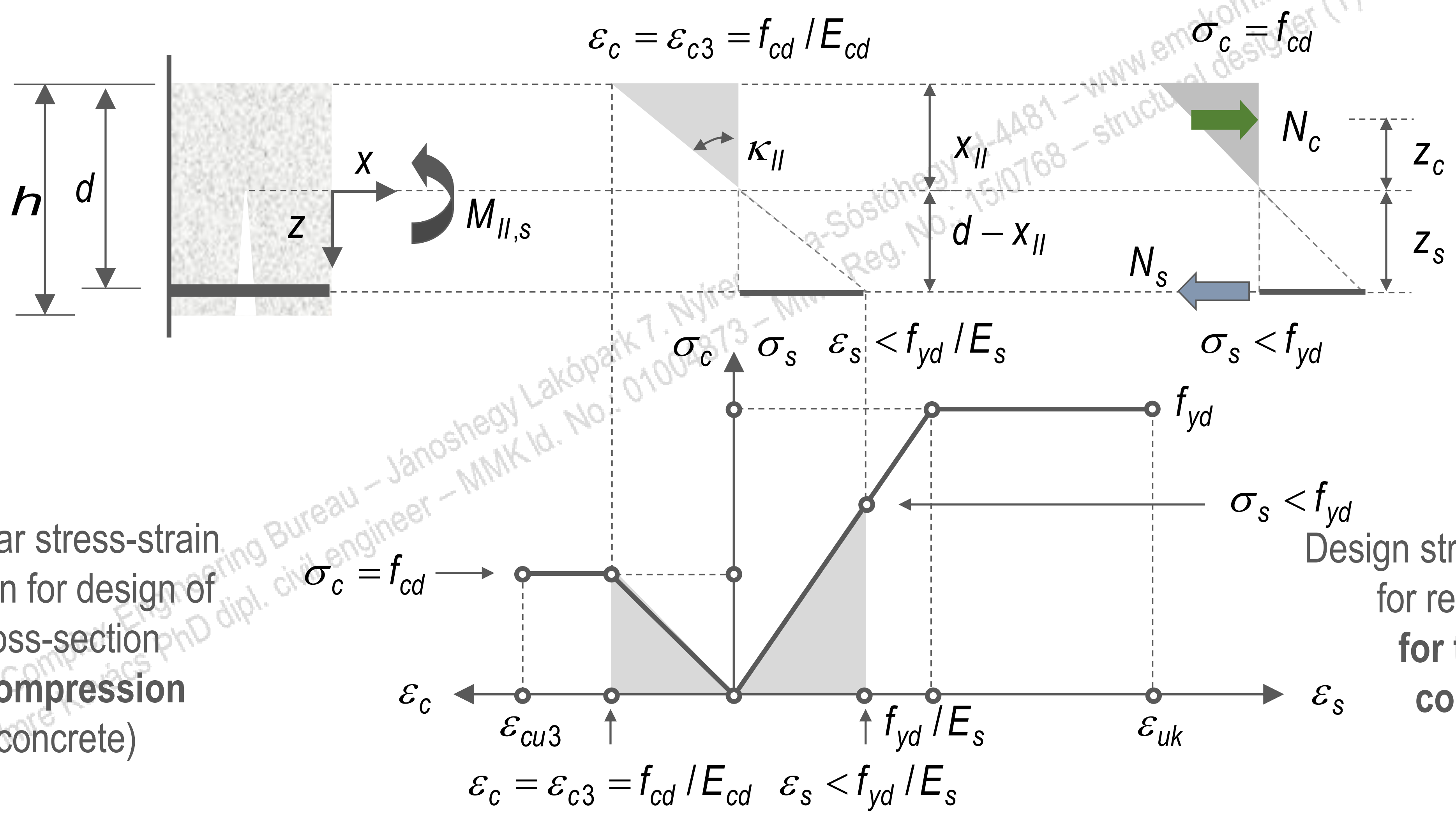
$$M_{II} = \frac{f_{yd}}{\alpha} \cdot \frac{I_{II}}{d - x_{II}} \quad \text{or} \quad M_{II} = \frac{f_{yd}}{\alpha} \cdot W_{II}^{s}$$

Concrete compressive stress and strain in the most compressed fibre of the cross-section corresponds to yielding of reinforcement:

$$\sigma_{c,II} = \frac{M_{II}}{I_{II}} \cdot x_{II} \qquad \varepsilon_{c,II} = \frac{\sigma_{c,II}}{E_{cd}}$$

The first plastic phenomenon is the crushing of the concrete – $\sigma_c = f_{cd}$

Geometry	Deformation, ε	Normal stress, σ

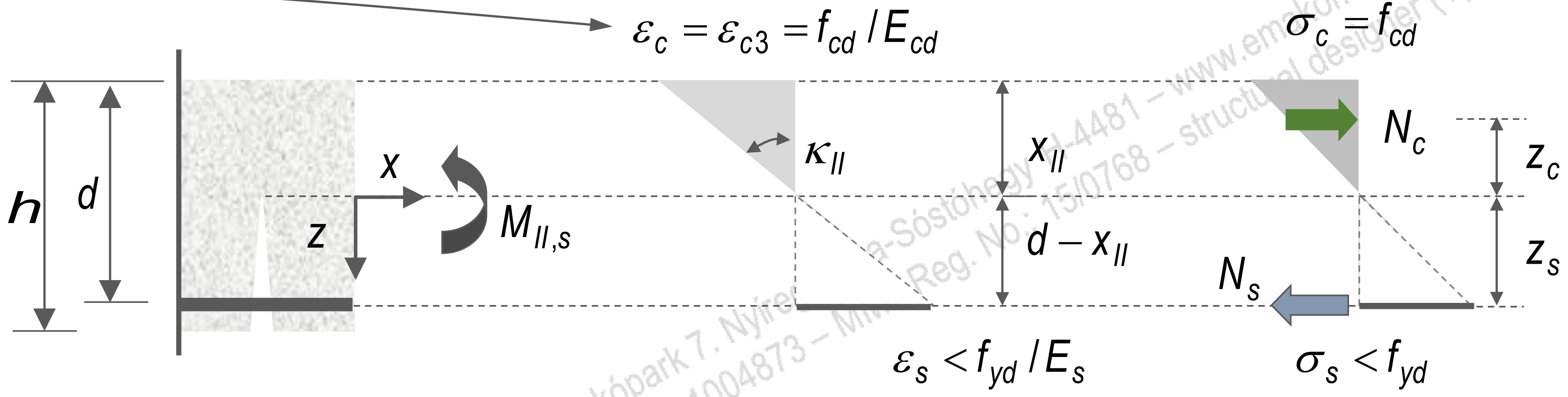


Bi-linear stress-strain relation for design of cross-section **in compression** (concrete)

Design stress-strain diagram for reinforcing steel **for tension and compression**

The first plastic phenomenon is the crushing of the concrete – $\sigma_c = f_{cd}$

Geometry	Deformation, ε	Normal stress, σ

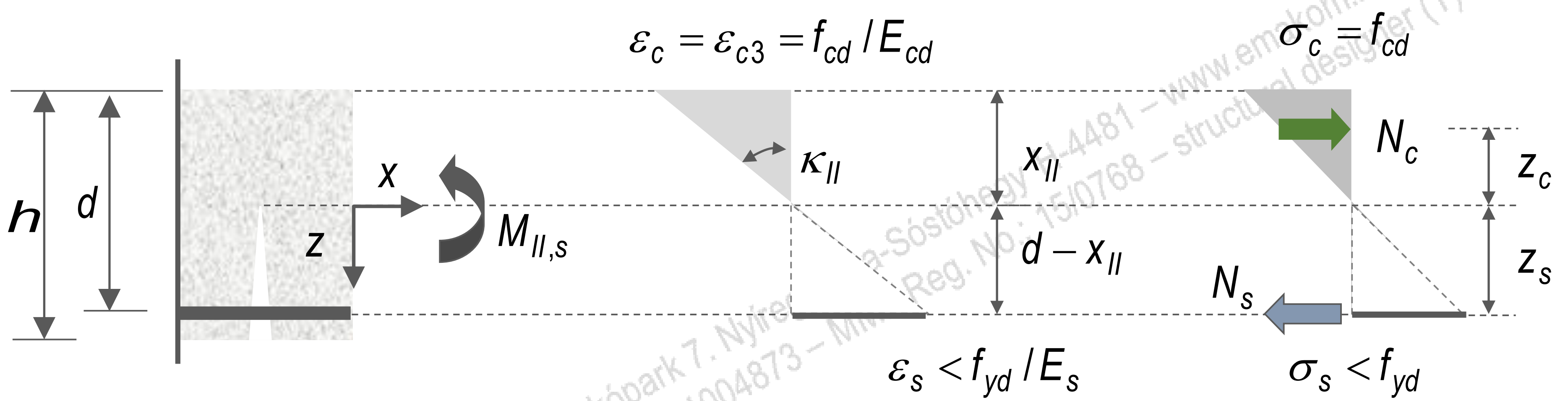


Bending moment corresponds to **crushing of concrete** – end of the elastic/cracked state of stress or end of the II. state of stress

$$\kappa_{II} = \frac{\varepsilon_c}{x_{II}} = \frac{f_{cd}}{E_{cd}} \cdot \frac{1}{x_{II}} \Rightarrow M_{II,c} = \kappa_{II} \cdot E_{cd} \cdot I_{II} = \frac{f_{cd}}{E_{cd}} \cdot E_{cd} \cdot \frac{\left\{ \frac{b \cdot x_{II}^3}{3} + \alpha \cdot A_s \cdot (d - x_{II})^2 \right\}}{x_{II}} = f_{cd} \cdot \frac{I_{II}}{x_{II}} = f_{cd} \cdot W_{II}^c$$

The first plastic phenomenon is the crushing of the concrete – $\sigma_c = f_{cd}$

Geometry	Deformation, ε	Normal stress, σ



Curveture corresponds to the crushing of concrete:

$$\kappa_{II} = \frac{f_{cd}}{E_{cd}} \cdot \frac{1}{x_{II}}$$

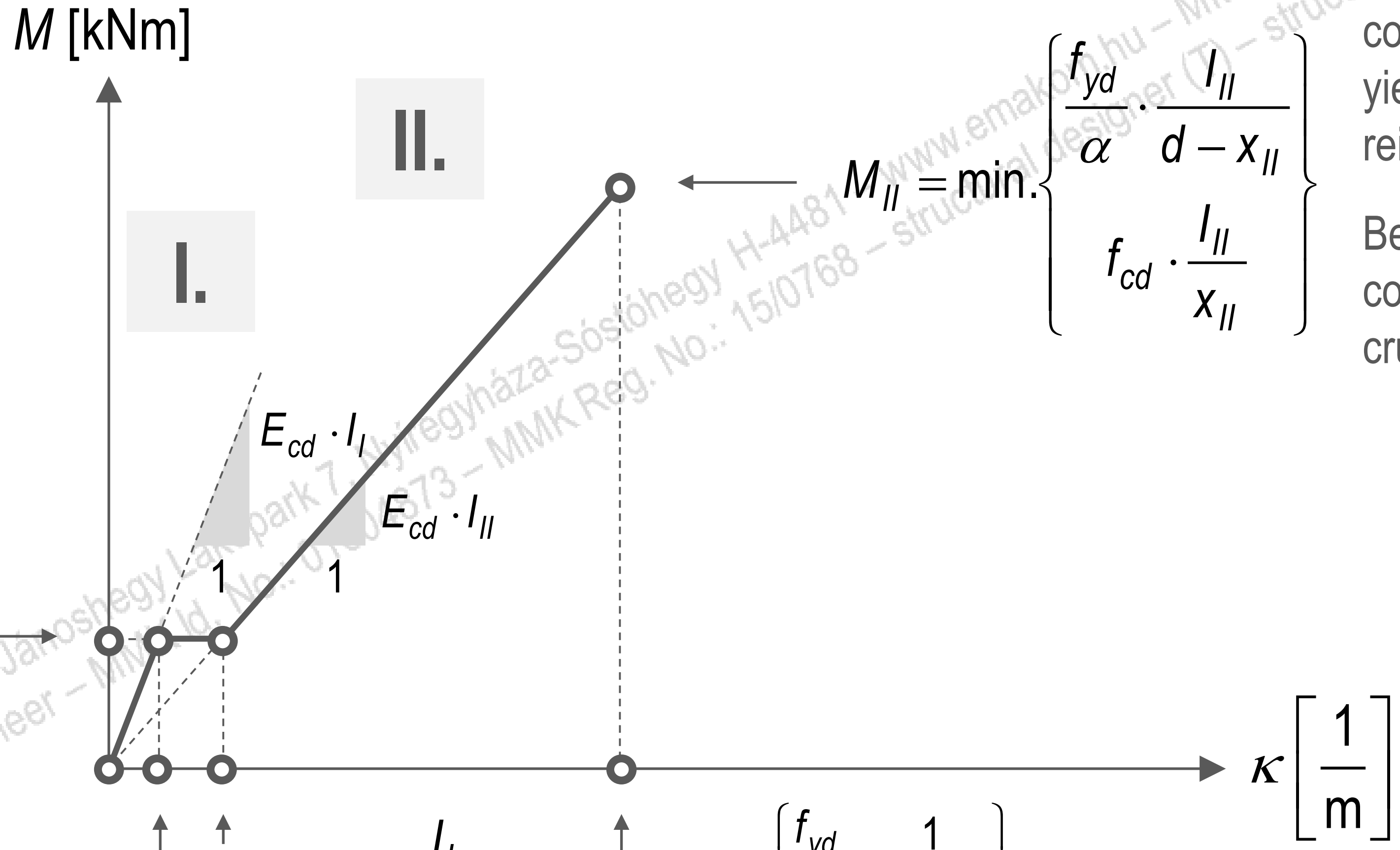
Bending moment corresponds to the crushing of concrete:

$$M_{II} = f_{cd} \cdot \frac{I_{II}}{x_{II}} \quad \text{or} \quad M_{II} = f_{cd} \cdot W_{II}^{c}$$

Reinforcement stress and strain at the central point of the tensioned reinforcement corresponds to crushing of concrete:

$$\sigma_{s,II} = \alpha \cdot \frac{M_{II}}{I_{II}} \cdot (d - x_{II}) \qquad \varepsilon_{s,II} = \frac{\sigma_{s,II}}{E_s}$$

RC cross-section in the uncracked and elastic/cracked state of stresses



$$M_{II} = \min.\begin{cases} \dfrac{f_{yd}}{\alpha} \cdot \dfrac{I_{II}}{d - x_{II}} \\ f_{cd} \cdot \dfrac{I_{II}}{x_{II}} \end{cases}$$

Bending moment corresponds to the yielding of reinforcement

Bending moment corresponds to the crushing of concrete

$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I}$$

$$M_I = M_{I-II} \rightarrow \kappa_I \cdot E_{cd} \cdot I_I = \kappa_{I-II} \cdot E_{cd} \cdot I_{II}$$

$$\kappa_{I-II} = \kappa_I \cdot \frac{I_I}{I_{II}}$$

$$\kappa_I = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h - x_I}$$

$$\kappa_{II} = \min.\begin{cases} \dfrac{f_{yd}}{E_s} \cdot \dfrac{1}{d - x_{II}} \\ \dfrac{f_{cd}}{E_{cd}} \cdot \dfrac{1}{x_{II}} \end{cases}$$

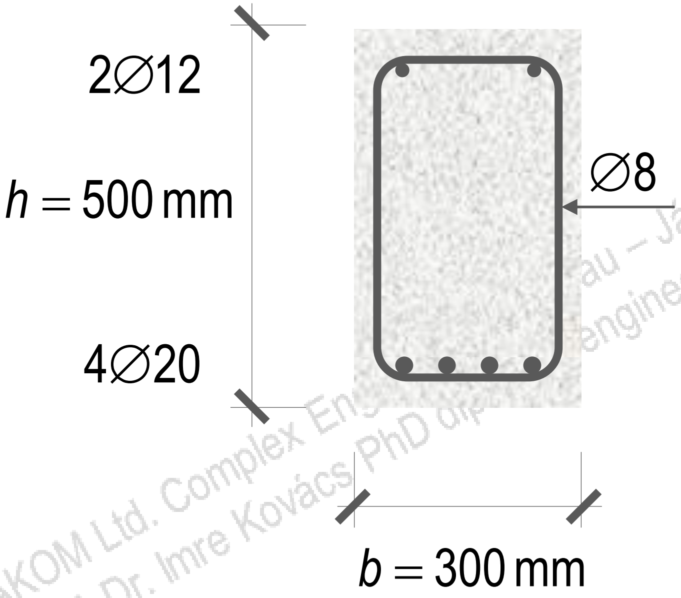
Curveture corresponds to the yielding of reinforcement

Curveture corresponds to the crushing of concrete:

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (1)

Determine the moment corresponds to the first plastic phenomenon of the outlined rectangular RC cross-section (**h = 500 mm, b = 300 mm**) in the persistent and transient design situation**,** if the concrete grade is **C30/37**, the nominal concrete cover on stirrups is $\boldsymbol{C_{nom}}$ **= 30 mm**, the maximum size of the aggregates is $\boldsymbol{d_g}$ **= 16 mm**, the diamater of the stirrup reinforcement is $\boldsymbol{Ø_s}$ **= 8 mm**, the tensioned reinforcement consists of **4Ø20,** the auxalary (compressed side) reinforcement consists of **2Ø12, B500A**! Determine and outline the strain and stress distribution of the cross-section at the first plastic phenomenon of the cross-section!

(See Topic 13. Example (1)!)



- Concrete grade: **C30/37**
- Aggregate: $\boldsymbol{d_g}$ **= 16 mm**
- RE bar grade: **B500A**
- Width of cross-section: ***b*** **= 300 mm**
- Height of cross-section: ***h*** **= 500 mm**
- Concrete cover: $\boldsymbol{C_{nom}}$ **= 30 mm**
- Tensioned reinforcement: $\boldsymbol{A_{s,prov}}$ **= 4Ø20 (1256 mm^2)**
- Auxalary reinforcement: $\boldsymbol{A_s'_{,prov}}$ **= 2Ø12**
- Stirrups: **Ø8**

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (1)

- $$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{30}{1{,}50} = 20\,\text{N/mm}^2$$ ☑
- $$E_{cd} = \frac{f_{cd}}{1{,}75\,\text{‰}} = \frac{20}{1{,}75} \approx 11400\,\text{N/mm}^2$$ ☑
- $$f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}15} = 435\,\text{N/mm}^2$$ ☑
- $$\alpha = E_s / E_{cd} = 200000/11400 \approx 17{,}5$$ ☑
  - $$\Delta\varnothing_{min} = \max\begin{Bmatrix} k_1 \cdot \varnothing = 1 \cdot 20 = 20\text{ mm} \\ d_g + k_2 = 16 + 5 = 21\text{ mm} \\ 20\text{ mm} \end{Bmatrix} = 21\text{ mm}$$ ☑
  - $$\Delta\varnothing = \frac{b - (2 \cdot C_{nom} + 2 \cdot \varnothing_s + n \cdot \varnothing)}{n-1} = \frac{300 - (2 \cdot 30 + 2 \cdot 8 + 4 \cdot 20)}{4-1} = 48\text{ mm} > \Delta\varnothing_{min} = 21\text{ mm}$$ ☑
  - $$d = h - \left(C_{nom} + \varnothing_s + \frac{\varnothing}{2}\right) = 500 - \left(30 + 8 + \frac{20}{2}\right) = 442\text{ mm}$$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (1)

- $0=\frac{1}{2}\cdot b\cdot x^2-(d-x)\cdot\alpha\cdot A_s$ ☑
- $a\cdot x^2+b\cdot x+c=0$
  $b\cdot x^2+(2\cdot\alpha\cdot A_s)\cdot x-2\cdot\alpha\cdot A_s\cdot d=0$ ☑

➡ $x=x_{II}=-\frac{\alpha\cdot A_s}{b}+\sqrt{\left(\frac{\alpha\cdot A_s}{b}\right)^2+\frac{\alpha\cdot A_s}{b}\cdot 2\cdot d}=-\frac{17{,}5\cdot 1256}{300}+\sqrt{\left(\frac{17{,}5\cdot 1256}{300}\right)^2+\frac{17{,}5\cdot 1256}{300}\cdot 2\cdot 442}=192\,\mathrm{mm}$ ☑

➡ $I_{II}=\frac{b\cdot x_{II}^{\,3}}{3}+\alpha\cdot A_{s,prov}\cdot(d-x_{II})^2=\frac{300\cdot 192^3}{3}+17{,}5\cdot 1256\cdot(442-192)^2=2{,}08\cdot 10^9\ \mathrm{mm}^4$ ☑

➡ $W_{II}^s=\frac{I_{II}}{d-x_{II}}=\frac{2{,}08\cdot 10^9}{442-192}=8{,}32\cdot 10^6\ \mathrm{mm}^3$ ☑

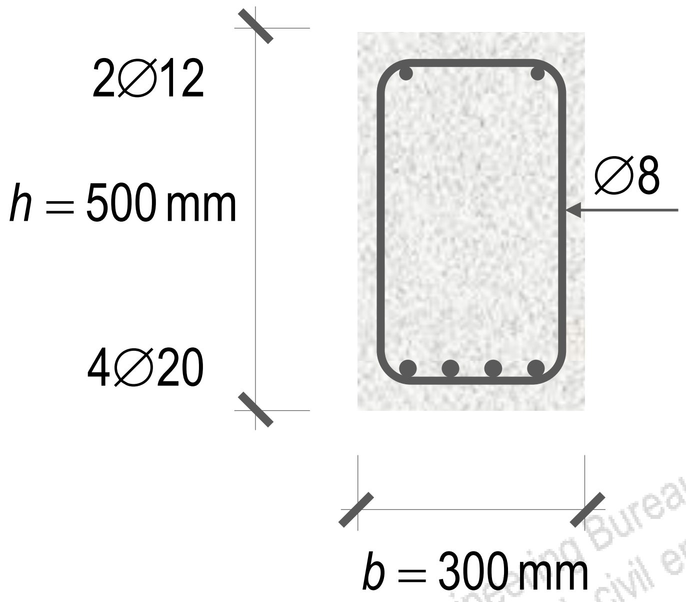
➡ $W_{II}^c=\frac{I_{II}}{x_{II}}=\frac{2{,}08\cdot 10^9}{192}=10{,}8\cdot 10^6\ \mathrm{mm}^3$ ☑

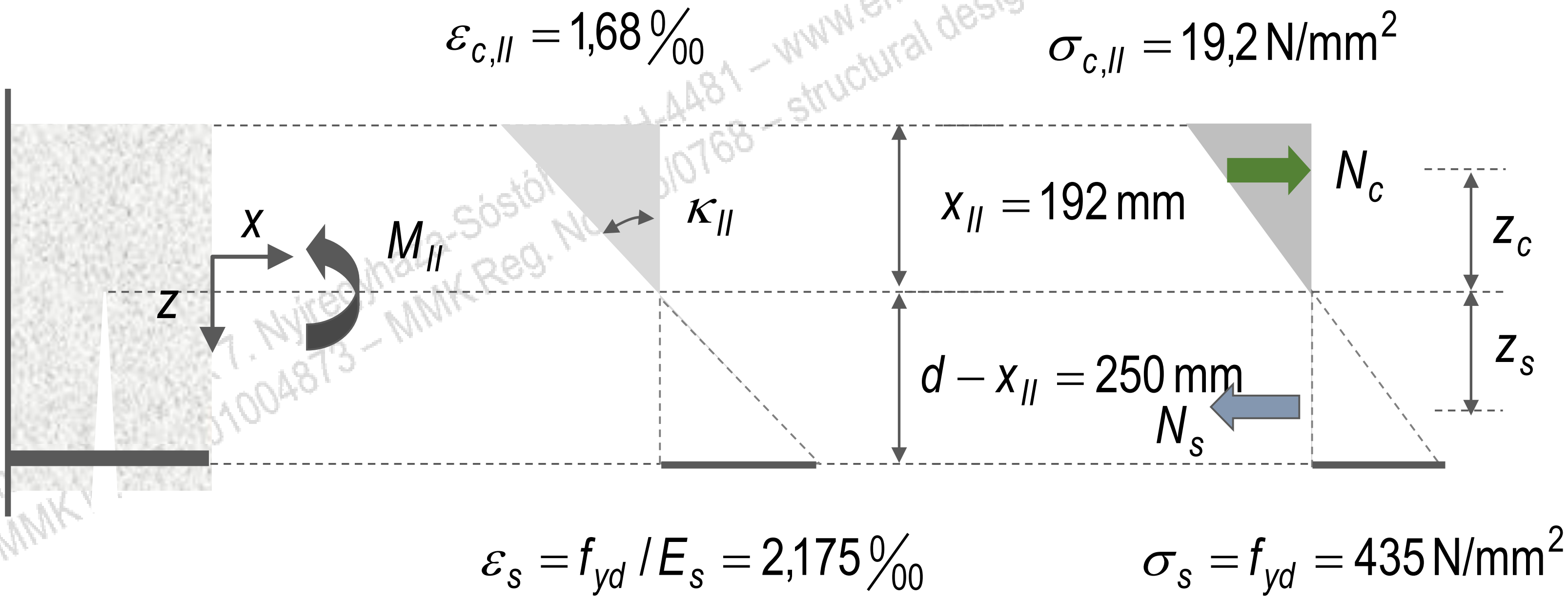
Example: Analysis of the RC cross-section in the elastic/cracked state of stress (1)

- $$M_{II} = \min \begin{Bmatrix} \frac{f_{yd}}{\alpha} \cdot \frac{I_{II}}{d - x_{II}} = \frac{f_{yd}}{\alpha} \cdot W_{II}^{s} = \frac{435}{17{,}5} \cdot 8{,}32 = 207\,\text{kNm} \\ f_{cd} \cdot \frac{I_{II}}{x_{II}} = f_{cd} \cdot W_{II}^{c} = 20 \cdot 10{,}8 \cdot 10^{6} = 216\,\text{kNm} \end{Bmatrix} = 207\,\text{kNm}$$ ☑
- The first plastic phenomenon is the yielding of the reinforcement! ☑
- $$\sigma_{c,II} = \frac{M_{II}}{I_{II}} \cdot x_{II} = \frac{M_{II}}{W_{II}^{c}} = \frac{207 \cdot 10^{6}}{10{,}8 \cdot 10^{6}} = 19{,}2\,\text{N/mm}^2 < f_{cd} = 20\,\text{N/mm}^2$$ ☑
- $$\varepsilon_{c,II} = \frac{\sigma_{c,II}}{E_{cd}} = \frac{19{,}2}{11400} = 1{,}68\,‰ < \varepsilon_{c3} = 1{,}75\,‰$$ ☑
- $$\kappa_{II} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}} = \frac{435}{200000} \cdot \frac{1}{422 - 192} = 9{,}46 \cdot 10^{-6}\,\frac{1}{\text{mm}}$$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (1)

Geometry | Deformation, ε | Normal stress, σ





$$\kappa_{II} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}} = \frac{435}{200000} \cdot \frac{1}{422 - 192} = 9{,}46 \cdot 10^{-6}\ \frac{1}{\text{mm}}$$

$$M_{II} = 207\ \text{kNm}$$

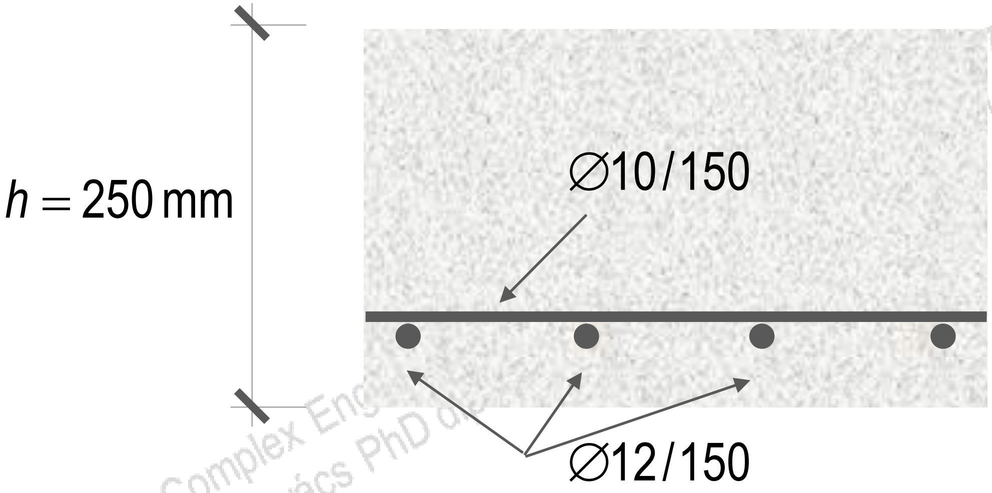
$$N_c = \frac{1}{2} \cdot b \cdot x_{II} \cdot \sigma_{c,II} = \frac{1}{2} \cdot 300 \cdot 192 \cdot 19{,}2 \approx 553\ \text{kN}$$

$$N_s = A_{s,prov} \cdot f_{yd} = 1256 \cdot 435 \approx 546\ \text{kN}$$

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (2)

Determine the moment corresponds to the first plastic phenomenon of the outlined RC slab cross-section (***h* = 250 mm**), in the persistent and transient design situation, if the concrete grade is **C25/30**, the nominal concrete cover is **C_{nom} = 25 mm**, the maximum size of the aggregates is **d_g = 24 mm**, the main reinforcement consists of **Ø12/150,** the transversal reinforcement consists of **Ø10/150, B500B**! Determine and outline the strain and stress distribution of the cross-section at the first plastic phenomenon of the cross-section!

(See Topic 13. Example (2)!)



- Concrete grade: **C25/30**
- Aggregate: **d_g = 24 mm**
- RE bar grade: **B500B**
- Height of slab: ***h* = 250 mm**
- Concrete cover: **C_{nom} = 25 mm**
- Main reinforcement: **$a_{s,prov}$ = Ø12/150 (754 mm^2)**
- Transversal reinforcement: **$a_{s,prov,trans}$ = Ø10/150**

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (2)

- $f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{25}{1{,}50} = 16{,}66\ \text{N/mm}^2$ ☑
- $E_{cd} = \frac{f_{cd}}{1{,}75\text{‰}} = \frac{16{,}66}{1{,}75} \approx 9500\ \text{N/mm}^2$ ☑
- $f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}15} = 435\ \text{N/mm}^2$ ☑
- $\alpha = E_s / E_{cd} = 200000/9500 \approx 21$ ☑
  - $d = h - \left(C_{nom} + \frac{\varnothing}{2}\right) = 250 - \left(25 + \frac{12}{2}\right) = 219\ \text{mm}$ ☑
  - $0 = \frac{1}{2} \cdot b \cdot x^2 - (d - x) \cdot \alpha \cdot a_s$ ☑
  - $a \cdot x^2 + b \cdot x + c = 0 \rightarrow b \cdot x^2 + (2 \cdot \alpha \cdot a_s) \cdot x - 2 \cdot \alpha \cdot a_s \cdot d = 0$ ☑
- $x = x_{II} = -\frac{\alpha \cdot a_s}{b} + \sqrt{\left(\frac{\alpha \cdot a_s}{b}\right)^2 + \frac{\alpha \cdot a_s}{b} \cdot 2 \cdot d} = -\frac{21 \cdot 754}{1000} + \sqrt{\left(\frac{21 \cdot 754}{1000}\right)^2 + \frac{21 \cdot 754}{1000} \cdot 2 \cdot 219} = 69\ \text{mm}$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (2)

- $$i_{II} = \frac{b \cdot x_{II}^{3}}{3} + \alpha \cdot a_{s,prov} \cdot (d - x_{II})^2 = \frac{1000 \cdot 69^3}{3} + 21 \cdot 754 \cdot (219 - 69)^2 = 0{,}466 \cdot 10^9 \text{ mm}^4/\text{m}$$ ☑

- $$w_{II}^{s} = \frac{i_{II}}{d - x_{II}} = \frac{0{,}466 \cdot 10^9}{219 - 69} = 3{,}11 \cdot 10^6 \text{ mm}^3/\text{m}$$ ☑

- $$w_{II}^{c} = \frac{i_{II}}{x_{II}} = \frac{0{,}466 \cdot 10^9}{69} = 6{,}75 \cdot 10^6 \text{ mm}^3/\text{m}$$ ☑

- $$m_{II} = \min \begin{Bmatrix} \frac{f_{yd}}{\alpha} \cdot \frac{i_{II}}{d - x_{II}} = \frac{f_{yd}}{\alpha} \cdot w_{II}^{s} = \frac{435}{21} \cdot 3{,}11 \cdot 10^6 = 64{,}40 \text{ kNm/m} \\ f_{cd} \cdot \frac{i_{II}}{x_{II}} = f_{cd} \cdot w_{II}^{c} = 16{,}66 \cdot 6{,}75 \cdot 10^6 = 112{,}50 \text{ kNm/m} \end{Bmatrix} = 64{,}40 \text{ kNm/m}$$ ☑

- The first plastic phenomenon is the yielding of the reinforcement! ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (2)

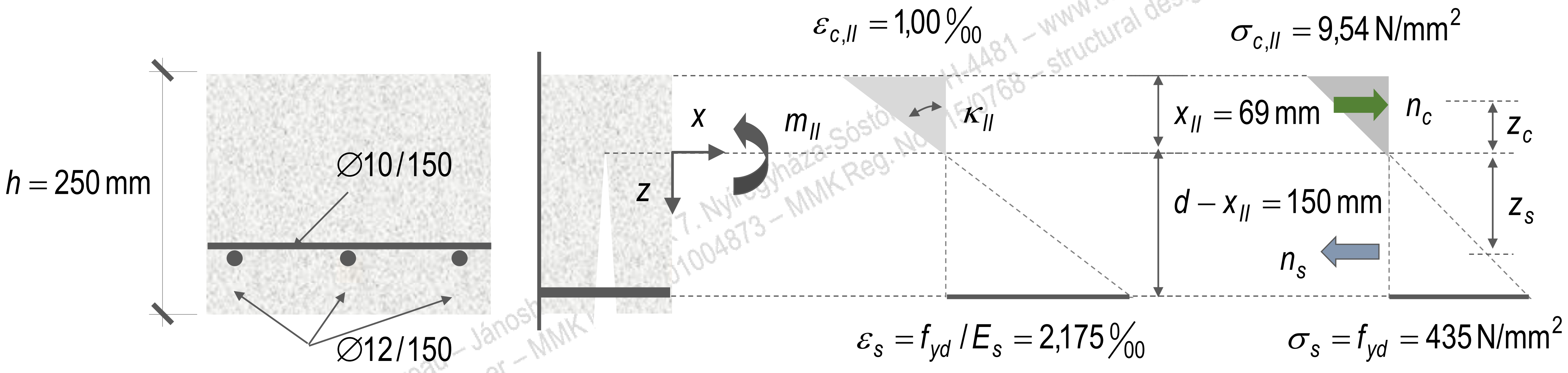
- $$\sigma_{c,II} = \frac{m_{II}}{i_{II}} \cdot x_{II} = \frac{m_{II}}{w_{II}^{c}} = \frac{64{,}40 \cdot 10^{6}}{6{,}75 \cdot 10^{6}} = 9{,}54\ \text{N/mm}^2 < f_{cd} = 16{,}66\ \text{N/mm}^2$$ ☑

- $$\varepsilon_{c,II} = \frac{\sigma_{c,II}}{E_{cd}} = \frac{9{,}54}{9500} = 1{,}00\ ‰ < \varepsilon_{c3} = 1{,}75\ ‰$$ ☑

- $$\kappa_{II} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}} = \frac{435}{200000} \cdot \frac{1}{219 - 69} = 14{,}50 \cdot 10^{-6}\ \frac{1}{\text{mm}}$$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (2)

Geometry | Deformation, ε | Normal stress, σ



$$\kappa_{II} = \frac{f_{yd}}{E_s} \cdot \frac{1}{d - x_{II}} = \frac{435}{200000} \cdot \frac{1}{219 - 69} = 14{,}50 \cdot 10^{-6}\,\frac{1}{\text{mm}}$$

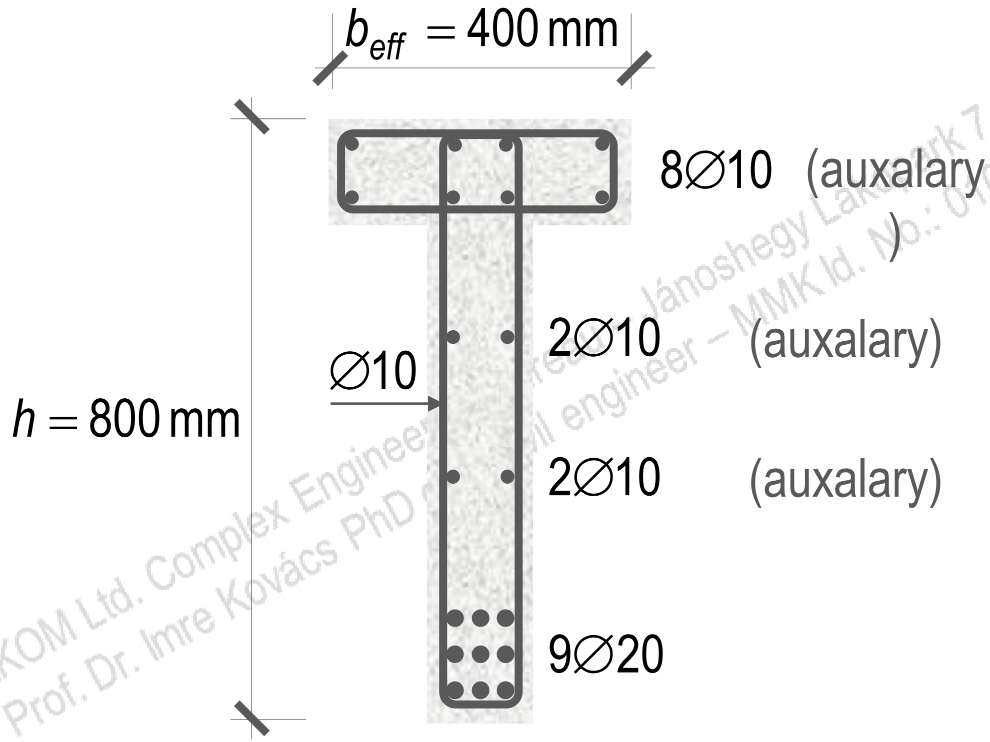
$$n_c = \frac{1}{2} \cdot b \cdot x_{II} \cdot \sigma_{c,II} = \frac{1}{2} \cdot 1000 \cdot 69 \cdot 9{,}54 \approx 329\,\text{kN/m}$$

$$m_{II} = 64{,}40\,\text{kNm/m}$$

$$n_s = a_{s,prov} \cdot f_{yd} = 754 \cdot 435 \approx 328\,\text{kN/m}$$

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

Determine the moment corresponds to the first plastic phenomenon of the outlined prefabricated RC ”T” cross-section (**h = 800 mm, b_{eff} = 400 mm, b_w = 160 mm, v = 160 mm**) in accidental design situation, if the concrete grade is **C50/60**, the nominal concrete cover on stirrups is **C_{nom} = 20 mm**, the maximum size of the aggregates is **d_g = 8 mm**, the diamater of the stirrup reinforcement is **$Ø_s$ = 10 mm**, the tensioned reinforcement consists of **9Ø20,** the auxalary (compressed side) reinforcement consists of **8Ø10, B500B**! Determine and outline the strain and stress distribution of the cross-section at the first plastic phenomenon of the cross-section!



(See Topic 13. Example (3)!)

- Concrete grade: **C50/60**
- Aggregate: **d_g = 8 mm**
- RE bar grade: **B500B**
- Height: **h = 800 mm**
- Width of flange: **b_{eff} = 400 mm**
- Width of web: **b_w = 160 mm**
- Thickness of flange: **v = 160 mm**
- Tensioned reinforcement: **$A_{s,prov}$ = 9Ø20 (2826 mm^2)**
- Auxalary reinforcement: **$A_s{}'_{,prov}$ = 12Ø10**
- Stirrups: **Ø10**

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

- $f_{cd} = \alpha_{cc} \cdot \dfrac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \dfrac{50}{1{,}20} = 41{,}66\ \text{N/mm}^2$ ☑
- $E_{cd} = \dfrac{f_{cd}}{1{,}75\ ‰} = \dfrac{41{,}66}{1{,}75} \approx 23800\ \text{N/mm}^2$ ☑
- $f_{yd} = \dfrac{f_{yk}}{\gamma_S} = \dfrac{500}{1{,}00} = 500\ \text{N/mm}^2$ ☑
- $\alpha = E_s / E_{cd} = 200000 / 23800 \approx 8{,}40$ ☑
  - $\Delta\varnothing_{\min} = \max\begin{Bmatrix} k_1 \cdot \varnothing = 1 \cdot 20 = 20\ \text{mm} \\ d_g + k_2 = 8 + 5 = 13\ \text{mm} \\ 20\ \text{mm} \end{Bmatrix} = 20\ \text{mm}$ ☑
  - $\Delta\varnothing = \dfrac{b_w - (2 \cdot C_{nom} + 2 \cdot \varnothing_s + n \cdot \varnothing)}{n-1} = \dfrac{160 - (2 \cdot 20 + 2 \cdot 10 + 3 \cdot 20)}{3-1} = 20\ \text{mm} = \Delta\varnothing_{\min} = 20\ \text{mm}$ ☑
  - $d = h - \left(C_{nom} + \varnothing_s + \varnothing + \Delta_{sor} + \dfrac{\varnothing}{2} \right) = 800 - \left(20 + 10 + 20 + 20 + \dfrac{20}{2} \right) = 720\ \text{mm}$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

- $0 = \frac{1}{2} \cdot b_{eff} \cdot x^2 - (d - x) \cdot \alpha \cdot A_s$ Let's consider first that the compressed belt is rectangular!!! ☑
- $a \cdot x^2 + b \cdot x + c = 0 \rightarrow b_{eff} \cdot x^2 + (2 \cdot \alpha \cdot A_s) \cdot x - 2 \cdot \alpha \cdot A_s \cdot d = 0$ ☑

$$a = b_{eff} = 400\,\text{mm}$$

$$b = 2 \cdot \alpha \cdot A_s = 2 \cdot 8{,}40 \cdot 2826 = 47467$$

$$c = -2 \cdot \alpha \cdot A_s \cdot d = -2 \cdot 8{,}40 \cdot 2826 \cdot 720 = -34183296$$

➡ $$x = x_{II} = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-47467 + \sqrt{47467^2 + 4 \cdot 400 \cdot 34183296}}{2 \cdot 400} = 239\,\text{mm} > v = 160\,\text{mm}$$ ☑

➡ Depth of the compressed belt is grater than the thickness of the flange!!! New horizontal force equilibrium equation is needed considering that the compressed belt is ”T” shaped! ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

Let's consider now that the compressed belt is "T" shaped!!!

- $0 = b_{eff} \cdot v \cdot \left(x - \frac{v}{2} \right) + b_w \cdot \frac{(x-v)^2}{2} - (d-x) \cdot \alpha \cdot A_s$ ☑
- $a \cdot x^2 + b \cdot x + c = 0 \rightarrow 0 = b_w \cdot x^2 + 2 \cdot (b_{eff} \cdot v - b_w \cdot v + \alpha \cdot A_s) \cdot x + b_w \cdot v^2 - b_{eff} \cdot v^2 - 2 \cdot \alpha \cdot A_s \cdot d$ ☑

$$a = b_w = 160\,\text{mm}$$

$$b = 2 \cdot (b_{eff} \cdot v - b_w \cdot v + \alpha \cdot A_s) = 2 \cdot (400 \cdot 160 - 160 \cdot 160 + 8{,}40 \cdot 2826) = 124277$$

$$c = b_w \cdot v^2 - b_{eff} \cdot v^2 - 2 \cdot d \cdot \alpha \cdot A_s = 160 \cdot 160^2 - 400 \cdot 160^2 - 2 \cdot 720 \cdot 8{,}40 \cdot 2826 = -40327296$$

➡ $x = x_{II} = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-124277 + \sqrt{124277^2 + 4 \cdot 160 \cdot 40327296}}{2 \cdot 160} = 246\,\text{mm} > v = 160\,\text{mm}$ ☑

➡ $I_{II} = \frac{b_{eff} \cdot v^3}{12} + (b_{eff} \cdot v) \cdot \left(x_{II} - \frac{v}{2} \right)^2 + \frac{b_w \cdot (x_{II} - v)^3}{3} + \alpha \cdot A_{s,prov} \cdot (d - x_{II})^2 =$ ☑

$$= \frac{400 \cdot 160^3}{12} + (400 \cdot 160) \cdot \left(246 - \frac{160}{2} \right)^2 + \frac{160 \cdot (246 - 160)^3}{3} + 8{,}40 \cdot 2826 \cdot (720 - 246)^2 = 7{,}27 \cdot 10^9\,\text{mm}^4$$

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

- $W_{II}^{s} = W_{II}^{s,2\,\text{line}} = \dfrac{I_{II}}{d_{2sor} - x_{II}} = \dfrac{7{,}27 \cdot 10^{9}}{720 - 246} = 15{,}30 \cdot 10^{6}\ \text{mm}^{3}$ ☑

- $W_{II}^{c} = \dfrac{I_{II}}{x_{II}} = \dfrac{7{,}27 \cdot 10^{9}}{246} = 29{,}60 \cdot 10^{6}\ \text{mm}^{3}$ ☑

- $M_{II} = \min \begin{Bmatrix} \dfrac{f_{yd}}{\alpha} \cdot \dfrac{I_{II}}{d - x_{II}} = \dfrac{f_{yd}}{\alpha} \cdot W_{II}^{s} = \dfrac{500}{8{,}40} \cdot 15{,}30 \cdot 10^{6} = 911\,\text{kNm} \\ f_{cd} \cdot \dfrac{I_{II}}{x_{II}} = f_{cd} \cdot W_{II}^{c} = 41{,}66 \cdot 29{,}60 \cdot 10^{6} = 1233\,\text{kNm} \end{Bmatrix} = 911\,\text{kNm}$ ☑

- The first plastic phenomenon is the yielding of the reinforcement! ☑

- $\sigma_{c,II} = \dfrac{M_{II}}{I_{II}} \cdot x_{II} = \dfrac{911 \cdot 10^{6}}{7{,}27 \cdot 10^{9}} \cdot 246 = 30{,}80\,\text{N/mm}^{2} < f_{cd} = 41{,}66\,\text{N/mm}^{2}$ ☑

- $\varepsilon_{c,II} = \dfrac{\sigma_{c}}{E_{cd}} = \dfrac{30{,}80}{23800} = 1{,}29\,‰ < \varepsilon_{c3} = 1{,}75\,‰$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

$$\sigma_c = \frac{M_{II}}{I_{II}} \cdot (x_{II} - v) = \frac{911 \cdot 10^6}{7{,}27 \cdot 10^9} \cdot (246 - 160) = 10{,}80\ \text{N/mm}^2$$

Concrete compressive stress at the bottom of the flange ☑

$$\varepsilon_c = \frac{\sigma_c}{E_{cd}} = \frac{10{,}80}{23800} = 0{,}454\ ‰$$

Concrete compressive strain at the bottom of the flange ☑

$$\frac{\varepsilon_{s,II,1line}}{d_{1line} - x_{II}} = \frac{\varepsilon_{c,II}}{x_{II}} \rightarrow \varepsilon_{s,II,1line} = \varepsilon_{c,II} \cdot \frac{d_{1line} - x_{II}}{x_{II}} = 1{,}29 \cdot \frac{760 - 246}{246} = 2{,}70\ ‰ > f_{yd} / E_s = 2{,}50\ ‰ \rightarrow \sigma_s = f_{yd}$$ ☑

$$\frac{\varepsilon_{s,II,2line}}{d_{2line} - x_{II}} = \frac{\varepsilon_{c,II}}{x_{II}} \rightarrow \varepsilon_{s,II,2line} = \varepsilon_{c,II} \cdot \frac{d_{2line} - x_{II}}{x_{II}} = 1{,}29 \cdot \frac{720 - 246}{246} = 2{,}50\ ‰ = f_{yd} / E_s = 2{,}50\ ‰ \rightarrow \sigma_s = f_{yd}$$ ☑

$$\frac{\varepsilon_{s,II,3line}}{d_{3line} - x_{II}} = \frac{\varepsilon_{c,II}}{x_{II}} \rightarrow \varepsilon_{s,II,3line} = \varepsilon_{c,II} \cdot \frac{d_{3line} - x_{II}}{x_{II}} = 1{,}29 \cdot \frac{680 - 246}{246} = 2{,}28\ ‰ < f_{yd} / E_s = 2{,}50\ ‰ \rightarrow \sigma_s < f_{yd}$$ ☑

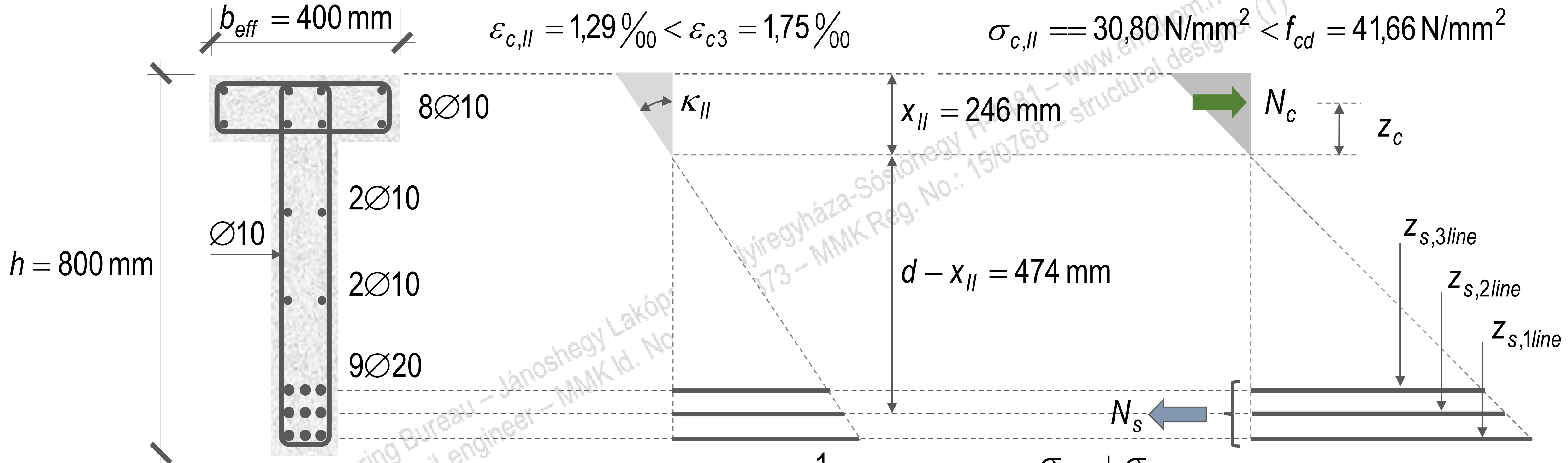
$$\sigma_{s,II,3line} = \varepsilon_{s,II,3line} \cdot E_s = 2{,}28\ ‰ \cdot 200000 = 456\ \text{N/mm}^2$$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (3)

Geometry	Deformation, ε	Normal stress, σ

$$\varepsilon_{c,II} = 1{,}29\,‰ < \varepsilon_{c3} = 1{,}75\,‰$$

$$\sigma_{c,II} == 30{,}80\,\mathrm{N/mm^2} < f_{cd} = 41{,}66\,\mathrm{N/mm^2}$$



$$N_c \approx \frac{1}{2} \cdot b_w \cdot x_{II} \cdot \sigma_{c,II} + \frac{\sigma_{c,II} + \sigma_c}{2} \cdot \left(b_{eff} - b_w\right) \cdot v$$

$$\approx \frac{1}{2} \cdot 160 \cdot 246 \cdot 30{,}80 + \frac{30{,}80 + 10{,}80}{2} \cdot (400 - 160) \cdot 160 \approx 1405\,\mathrm{kN}$$

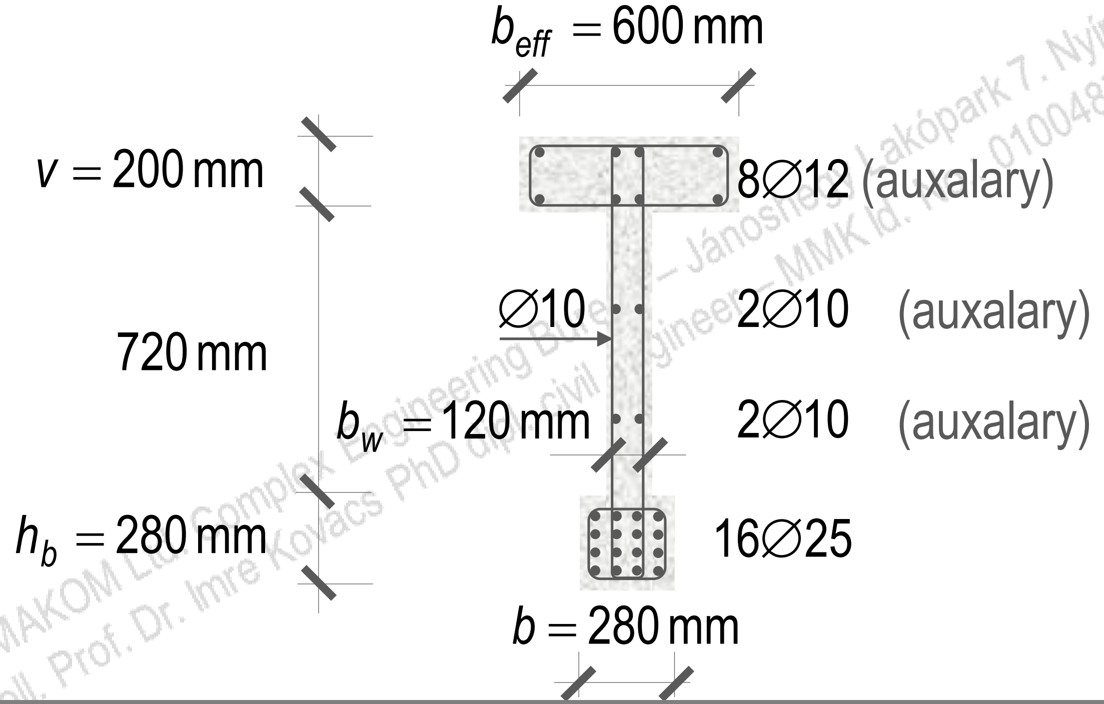
$$N_s \approx A_{s,prov} \cdot f_{yd} \approx 2826 \cdot 500 \approx 1413\,\mathrm{kN}$$

$$\kappa_{II} = \frac{\varepsilon_{c,II}}{x_{II}} = \frac{1{,}29}{246} = 5{,}24 \cdot 10^{-6}\ \mathrm{1/mm}$$

$$M_{II} = 911\,\mathrm{kNm}$$

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

Determine the moment corresponds to the first plastic phenomenon of the outlined prefabricated RC ”I” cross-section (**h = 1200 mm, b_{eff} = 600 mm, b_w = 120 mm, v = 200 mm, b = 280 mm, h_b = 280 mm**) in seismic design situation, if the concrete grade is **C40/50**, the nominal concrete cover on stirrups is **C_{nom} = 20 mm**, the maximum size of the aggregates is **d_g = 8 mm**, the diamater of the stirrup reinforcement is **$Ø_s$ = 10 mm**, the tensioned reinforcement consists of **16Ø25,** the auxalary (compressed side) reinforcement consists of **4Ø10+ 8Ø12, B500B**! Determine and outline the strain and stress distribution of the cross-section at the first plastic phenomenon of the cross-section!



(See Topic 13. Example (4)!)

- Concrete grade: **C40/50**
- Aggregate: **d_g = 8 mm**
- Re bar grade: **B500B**
- Height: **h = 1200 mm**
- Width of top flange: **b_{eff} = 600 mm**
- Width of web: **b_w = 120 mm**
- Thickness of top flange: **v = 200 mm**
- Width of bottom flange: **b = 280 mm**
- Thickness of bottom flange: **h_b = 280 mm**
- Main reinforcement **$A_{s,prov}$ = 16Ø25 (7856 mm^2)**
- Auxalary reinforcement: **$A_{s}'_{,prov}$ = 4Ø10+8Ø12**
- Stirrups: **Ø10**

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

- $$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{40}{1{,}20} = 33{,}33\ \text{N/mm}^2$$ ☑
- $$E_{cd} = \frac{f_{cd}}{1{,}75\,‰} = \frac{33{,}33}{1{,}75} \approx 19000\ \text{N/mm}^2$$ ☑
- $$f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}00} = 500\ \text{N/mm}^2$$ ☑
- $$\alpha = E_s / E_{cd} = 200000 / 19000 \approx 10{,}50$$ ☑
  - $$\Delta\varnothing_{min} = \max\begin{Bmatrix} k_1 \cdot \varnothing = 1 \cdot 25 = 25\ \text{mm} \\ d_g + k_2 = 8 + 5 = 13\ \text{mm} \\ 20\ \text{mm} \end{Bmatrix} = 25\ \text{mm}$$ ☑
  - $$\Delta\varnothing = \frac{b - (2 \cdot C_{nom} + 2 \cdot \varnothing_s + n \cdot \varnothing)}{n - 1} = \frac{280 - (2 \cdot 20 + 2 \cdot 10 + 4 \cdot 25)}{4 - 1} = 40\ \text{mm} > \Delta\varnothing_{min} = 25\ \text{mm}$$ ☑
  - $$d = h - \left(C_{nom} + \varnothing_s + \varnothing + \Delta_{sor} + \varnothing + \frac{\Delta_{sor}}{2} \right) = 1200 - \left(20 + 10 + 20 + 40 + 20 + \frac{40}{2} \right) = 1070\ \text{mm}$$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

- $0 = \frac{1}{2} \cdot b_{eff} \cdot x^2 - (d - x) \cdot \alpha \cdot A_s$ Let's consider first that the compressed belt is rectangular!!! ☑
- $a \cdot x^2 + b \cdot x + c = 0 \rightarrow b_{eff} \cdot x^2 + (2 \cdot \alpha \cdot A_s) \cdot x - 2 \cdot \alpha \cdot A_s \cdot d = 0$ ☑

$$a = b_{eff} = 600\,\text{mm}$$

$$b = 2 \cdot \alpha \cdot A_s = 2 \cdot 10{,}50 \cdot 7856 = 164976$$

$$c = -2 \cdot \alpha \cdot A_s \cdot d = -2 \cdot 10{,}50 \cdot 7856 \cdot 1070 = -176524320$$

➡ $$x = x_{II} = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-164976 + \sqrt{164976^2 + 4 \cdot 600 \cdot 176524320}}{2 \cdot 600} = 422\,\text{mm} > \text{v} = 200\,\text{mm}$$ ☑

➡ Depth of the compressed belt is grater than the thickness of the top flange!!! New horizontal force equilibrium equation is needed considering that the compressed belt is ”T” shaped! ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

- $$0 = b_{eff} \cdot v \cdot \left(x - \frac{v}{2} \right) + b_w \cdot \frac{(x - v)^2}{2} - (d - x) \cdot \alpha \cdot A_s$$ Let's consider now that the compressed belt is "T" shaped!!! ☑
- $$a \cdot x^2 + b \cdot x + c = 0 \rightarrow 0 = b_w \cdot x^2 + 2 \cdot (b_{eff} \cdot v - b_w \cdot v + \alpha \cdot A_s) \cdot x + b_w \cdot v^2 - b_{eff} \cdot v^2 - 2 \cdot \alpha \cdot A_s \cdot d$$ ☑

$$a = b_w = 120\,\text{mm}$$

$$b = 2 \cdot (b_{eff} \cdot v - b_w \cdot v + \alpha \cdot A_s) = 2 \cdot (600 \cdot 200 - 120 \cdot 200 + 10{,}50 \cdot 7856) = 356976$$

$$c = b_w \cdot v^2 - b_{eff} \cdot v^2 - 2 \cdot d \cdot \alpha \cdot A_s = 120 \cdot 200^2 - 600 \cdot 200^2 - 2 \cdot 1070 \cdot 10{,}50 \cdot 7856 = -195724320$$

➡ $$x = x_{II} = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-356976 + \sqrt{356976^2 + 4 \cdot 120 \cdot 195724320}}{2 \cdot 120} = 473\,\text{mm} > v = 200\,\text{mm}$$ ☑

➡ $$I_{II} = \frac{b_{eff} \cdot v^3}{12} + (b_{eff} \cdot v) \cdot \left(x_{II} - \frac{v}{2} \right)^2 + \frac{b_w \cdot (x_{II} - v)^3}{3} + \alpha \cdot A_{s,prov} \cdot (d - x_{II})^2 =$$ ☑

$$= \frac{600 \cdot 200^3}{12} + (600 \cdot 200) \cdot \left(473 - \frac{200}{2} \right)^2 + \frac{120 \cdot (473 - 200)^3}{3} + 10{,}50 \cdot 7856 \cdot (1070 - 473)^2 = 47{,}30 \cdot 10^9\,\text{mm}^4$$

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

$$W_{II}^{s} = \frac{I_{II}}{d - x_{II}} = \frac{47{,}30 \cdot 10^{9}}{1070 - 473} = 79{,}20 \cdot 10^{6}\ \text{mm}^{3}$$ ☑

$$W_{II}^{c} = \frac{I_{II}}{x_{II}} = \frac{47{,}30 \cdot 10^{9}}{473} = 100 \cdot 10^{6}\ \text{mm}^{3}$$ ☑

$$M_{II} = \min \left\{ \begin{matrix} \frac{f_{yd}}{\alpha} \cdot \frac{I_{II}}{d - x_{II}} = \frac{f_{yd}}{\alpha} \cdot W_{II}^{s} = \frac{500}{10{,}50} \cdot 79{,}20 \cdot 10^{6} = 3771\,\text{kNm} \\ f_{cd} \cdot \frac{I_{II}}{x_{II}} = f_{cd} \cdot W_{II}^{c} = 33{,}33 \cdot 100 \cdot 10^{6} = 3333\,\text{kNm} \end{matrix} \right\} = 3333\,\text{kNm}$$ ☑

The first plastic phenomenon is the crushing of the concrete! ☑

$$\sigma_c = \frac{M_{II}}{I_{II}} \cdot (x_{II} - v) = \frac{3333 \cdot 10^{6}}{47{,}30 \cdot 10^{9}} \cdot (473 - 200) = 19{,}20\,\text{N/mm}^2$$

Concrete compressive stress at the bottom of the top flange ☑

$$\varepsilon_c = \frac{\sigma_c}{E_{cd}} = \frac{19{,}20}{19000} = 1{,}01\text{‰}$$

Concrete compressive strain at the bottom of the top flange ☑

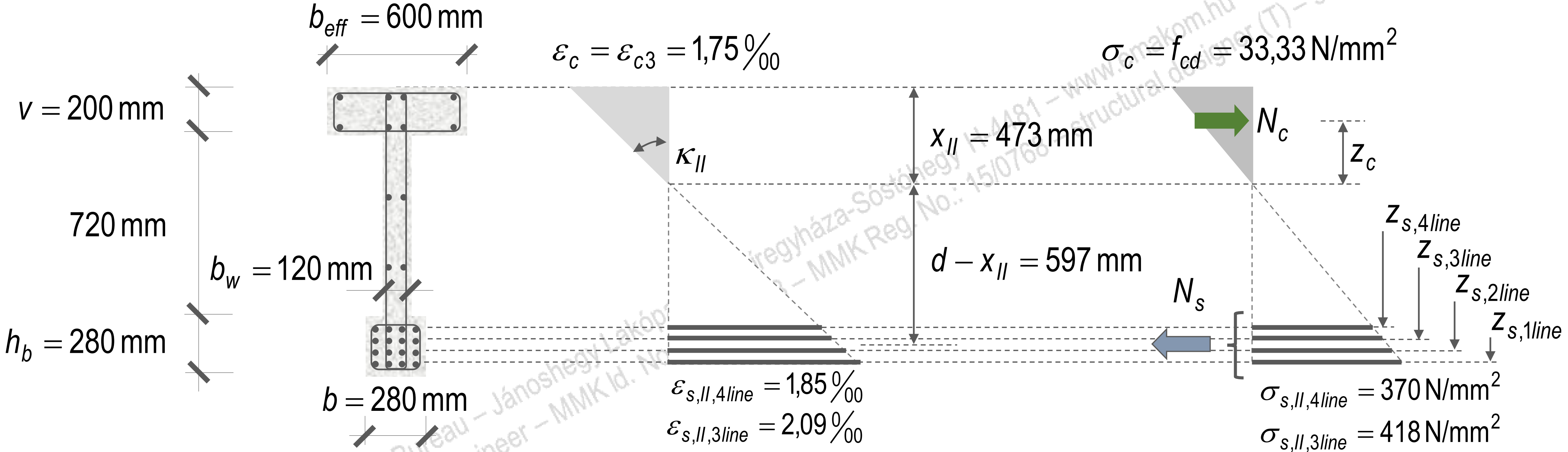
Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

- $\frac{\varepsilon_{s,II,1line}}{d_{1line} - x_{II}} = \frac{\varepsilon_{c3}}{x_{II}} \rightarrow \varepsilon_{s,II,1line} = \varepsilon_{c3} \cdot \frac{d_{1line} - x_{II}}{x_{II}} = 1{,}75 \cdot \frac{1167 - 473}{473} = 2{,}57\,‰ > f_{yd} / E_s = 2{,}50\,‰ \rightarrow \sigma_s = f_{yd}$ ☑
- $\frac{\varepsilon_{s,II,2line}}{d_{2line} - x_{II}} = \frac{\varepsilon_{c3}}{x_{II}} \rightarrow \varepsilon_{s,II,2line} = \varepsilon_{c3} \cdot \frac{d_{2line} - x_{II}}{x_{II}} = 1{,}75 \cdot \frac{1102 - 473}{473} = 2{,}33\,‰ < f_{yd} / E_s = 2{,}50\,‰ \rightarrow \sigma_s < f_{yd}$ ☑
- $\frac{\varepsilon_{s,II,3line}}{d_{3line} - x_{II}} = \frac{\varepsilon_{c3}}{x_{II}} \rightarrow \varepsilon_{s,II,3line} = \varepsilon_{c3} \cdot \frac{d_{3line} - x_{II}}{x_{II}} = 1{,}75 \cdot \frac{1037 - 473}{473} = 2{,}09\,‰ < f_{yd} / E_s = 2{,}50\,‰ \rightarrow \sigma_s < f_{yd}$ ☑
- $\frac{\varepsilon_{s,II,4line}}{d_{4line} - x_{II}} = \frac{\varepsilon_{c3}}{x_{II}} \rightarrow \varepsilon_{s,II,4line} = \varepsilon_{c4} \cdot \frac{d_{4line} - x_{II}}{x_{II}} = 1{,}75 \cdot \frac{972 - 473}{473} = 1{,}85\,‰ < f_{yd} / E_s = 2{,}50\,‰ \rightarrow \sigma_s < f_{yd}$ ☑

- $\sigma_{s,II,2line} = \varepsilon_{s,II,2line} \cdot E_s = 2{,}33\,‰ \cdot 200000 = 466\,\text{N/mm}^2$ ☑
- $\sigma_{s,II,3line} = \varepsilon_{s,II,3line} \cdot E_s = 2{,}09\,‰ \cdot 200000 = 418\,\text{N/mm}^2$ ☑
- $\sigma_{s,II,4line} = \varepsilon_{s,II,4line} \cdot E_s = 1{,}85\,‰ \cdot 200000 = 370\,\text{N/mm}^2$ ☑

Example: Analysis of the RC cross-section in the elastic/cracked state of stress (4)

Geometry	Deformation, ε	Normal stress, σ



$$\kappa_{II} = \frac{\varepsilon_{c3}}{x_{II}} = \frac{1{,}75}{473} = 3{,}70 \cdot 10^{-6}\ {}^{1}\!/\!_{mm}$$

$$N_s \approx \sum_{i=1}^{4} A_{s,prov,iline} \cdot \sigma_{s,iline} \approx 3445\,kN$$

$$M_{II} = 3333\,kNm$$

$$N_c \approx \frac{1}{2} \cdot b_w \cdot x_{II} \cdot f_{cd} + \frac{f_{cd} + \sigma_c}{2} \cdot (b_{eff} - b_w) \cdot v \approx \frac{1}{2} \cdot 120 \cdot 473 \cdot 33{,}33 + \frac{33{,}33 + 19{,}20}{2} \cdot (600 - 120) \cdot 200 \approx 3467\,kN$$

Reinforced Concrete (RC) Structures

Topic 14.

Cracked state of stress - II. state of stress

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Thank you for your kind attantion!

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