Reinforced Concrete (RC) Structures

Topic 13.

Uncracked state of stress - I. state of stress

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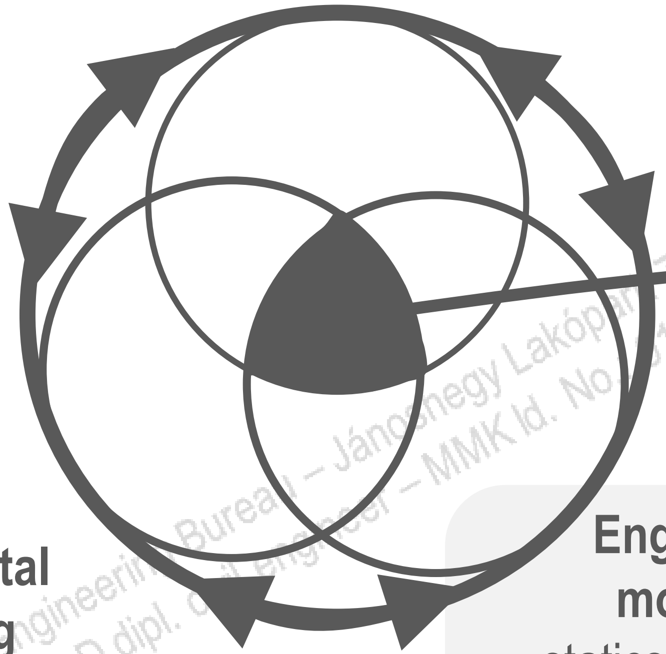
Modeling of structural behaviour of RC members

Numerical modelling
linear, non linear, plastic, 1st order theory, 2nd order theory, etc.

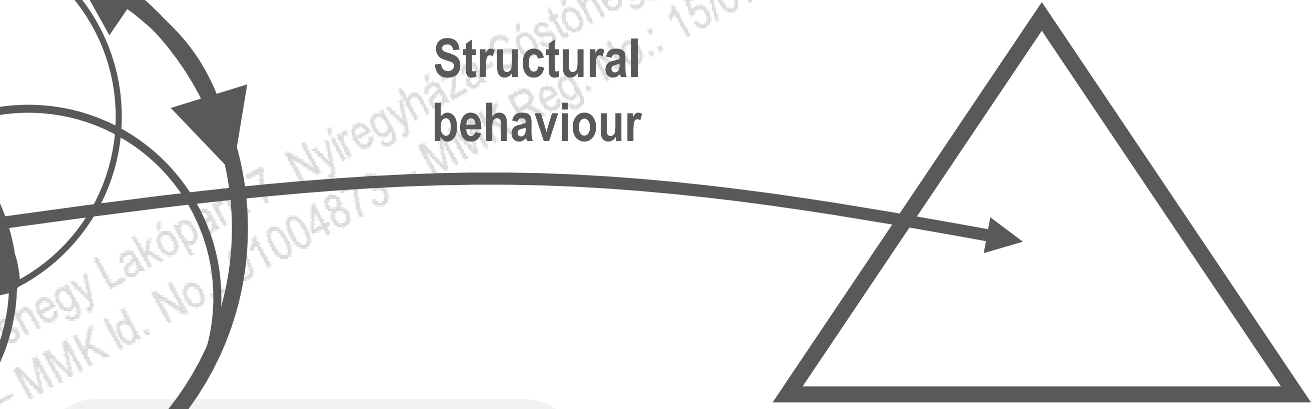


Experimental modelling
full-scale,
non full-scale, etc.

Engineering modelling
statics, strength of materials, elasticity, plasticity, dynamics, etc.

Structural behaviour

Material properties
homogen, inhomogen, izotropic, anizotropic
elastic linear, non linear elastic,
plastic, viscous, reological properties, etc.

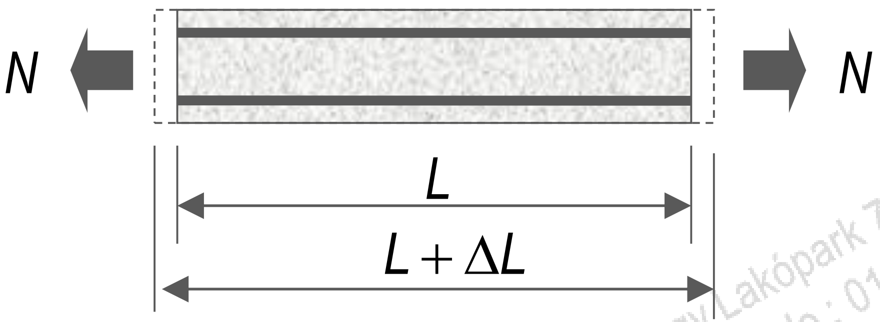


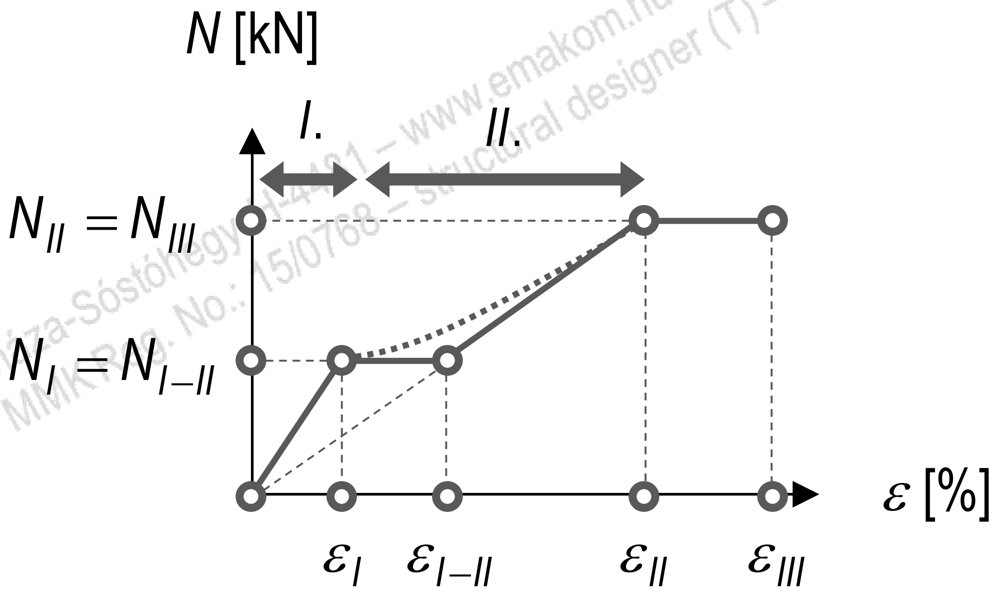
Size effect

Environment
effects, loads,
durability properties, etc.

Behaviour of reinforced concrete member in tension

Tension





$$\varepsilon = \frac{\Delta L}{L}$$

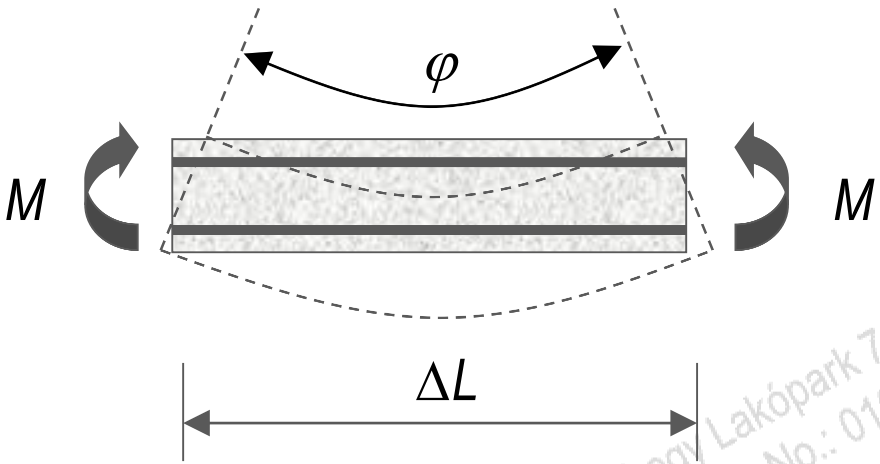
Deformation (strain)

$$\Delta L_I = \frac{\Delta N \cdot L}{E_{cd} \cdot A_i} \qquad \Delta L_{II} = \frac{\Delta N \cdot L}{E_s \cdot A_s}$$

Normal stiffness

Behaviour of reinforced concrete member in bending

Bending



$$\varphi = \frac{M}{E \cdot I} \cdot \Delta L \qquad \kappa = \frac{M}{E \cdot I}$$

Rotation | Curvature

$q(x)$ …load…

$V(x) = \int q(x) \cdot dx$ …shear force…

$M(x) = \int V(x) \cdot dx$ …bending moment…

$\varphi(x) = \int M(x) \cdot dx$ …rotation…

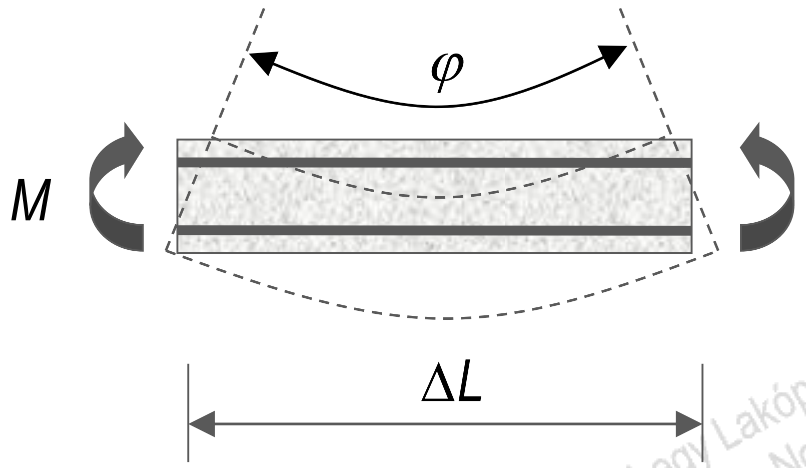
$v(x) = \int \varphi(x) \cdot dx$ …displacement…

$\kappa(x) = -\frac{d^2 v(x)}{dx^2}$ …curvature…

$v(x) = \int \left(\int \kappa(x) \cdot dx \right) \cdot dx$ …displacement…

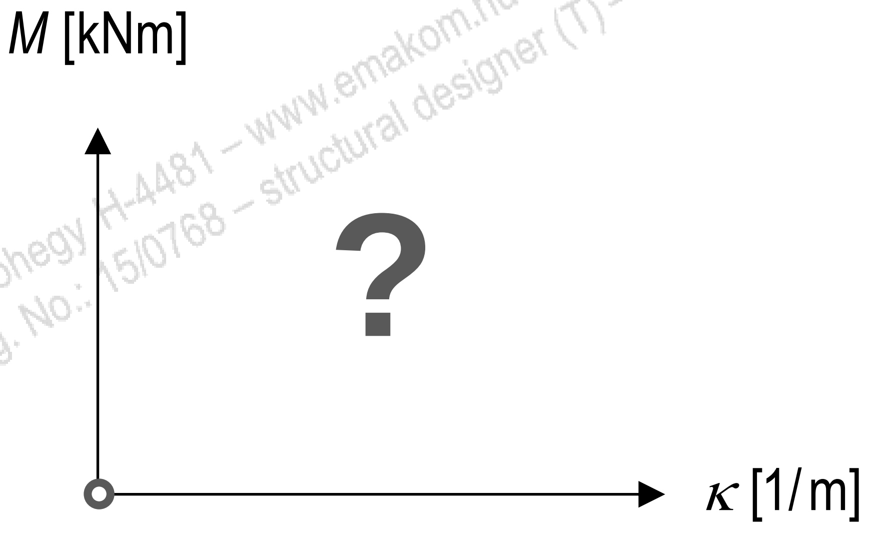
Deformation of reinforced concrete member in bending

Bending



$$\varphi = \frac{M}{E \cdot I} \cdot \Delta L \qquad \kappa = \frac{M}{E \cdot I}$$

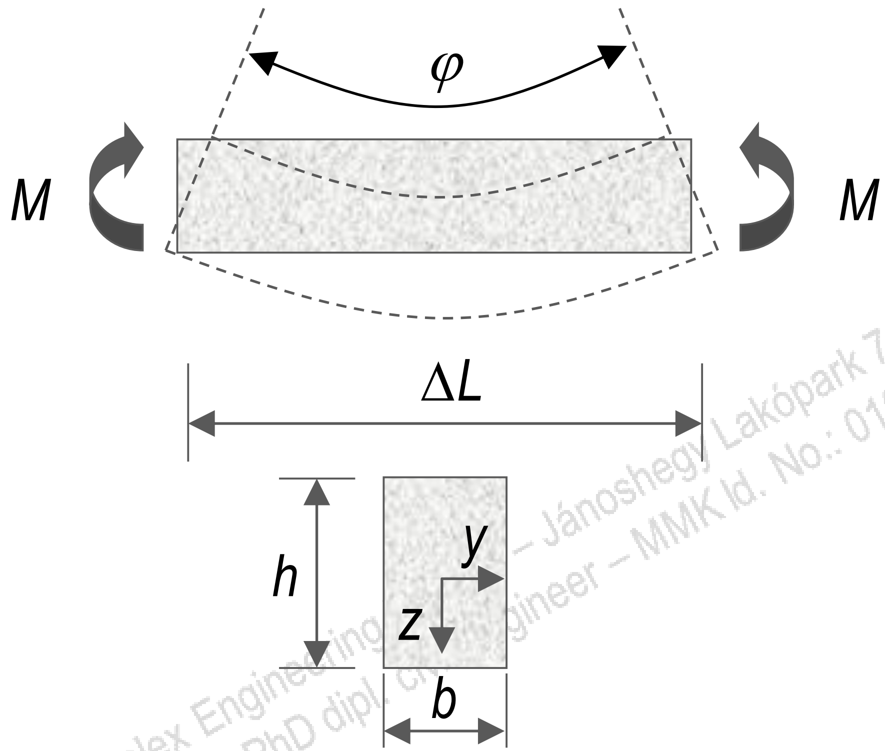
Rotation Curvature



The load process of a reinforced concrete member in bending can be examined using the **Bending moment - Curvature** relationship!

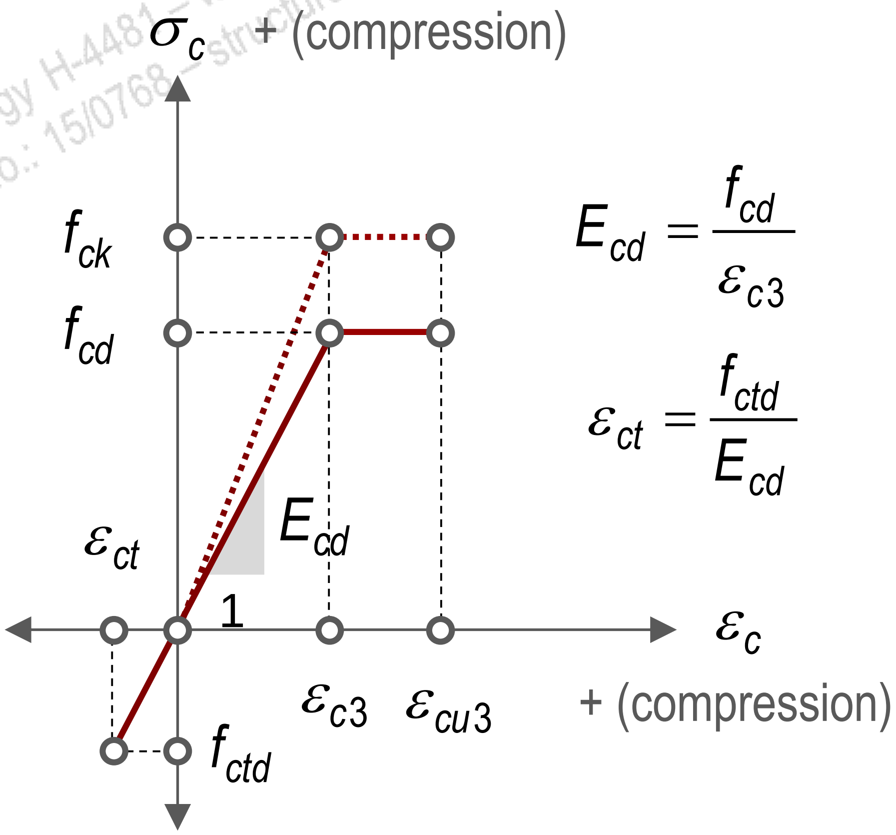
Load process of concrete member in bending – elastic state

Geometry



Cross-section

Material properties



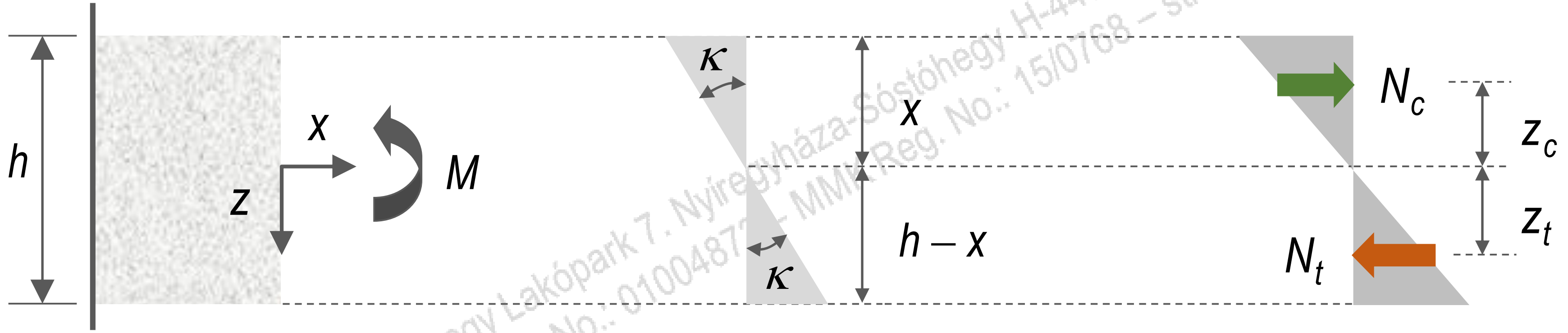
$$E_{cd} = \frac{f_{cd}}{\varepsilon_{c3}}$$

$$\varepsilon_{ct} = \frac{f_{ctd}}{E_{cd}}$$

Load process of concrete member in bending – elastic state

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

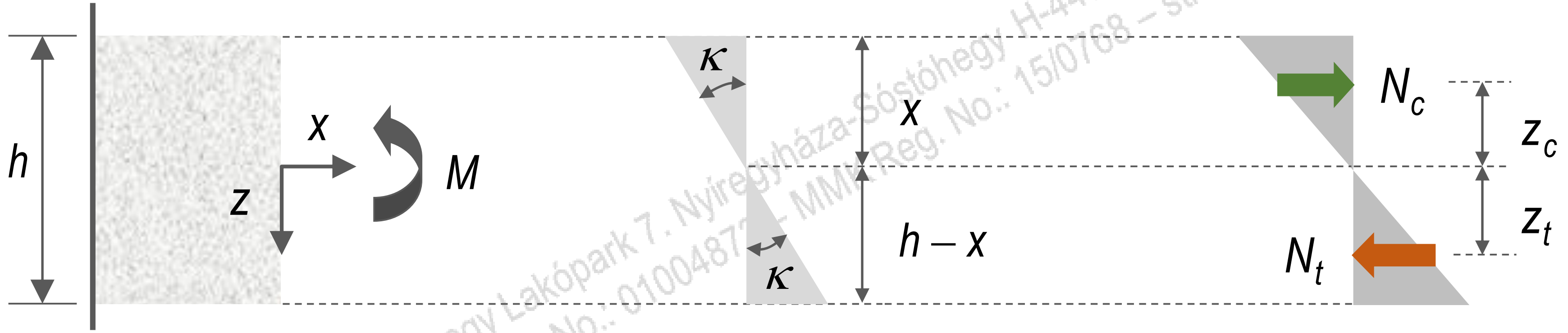
1. Horizontal force equilibrium: $\Sigma N = 0 \rightarrow 0 = N_c - N_t \rightarrow x = \dfrac{h}{2}$

2. Moment equilibrium: $\Sigma M = 0 \rightarrow 0 = M - N_c \cdot z_c - N_t \cdot z_t$

Load process of concrete member in bending – elastic state

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

2. Moment equilibrium: $M = N_c \cdot z_c + N_t \cdot z_t$

Internal forces: $N_c = \frac{1}{2} \cdot (\kappa \cdot x \cdot E_{cd}) \cdot (x \cdot b)$

$N_t = \frac{1}{2} \cdot (\kappa \cdot [h - x] \cdot E_{cd}) \cdot ([h - x] \cdot b)$

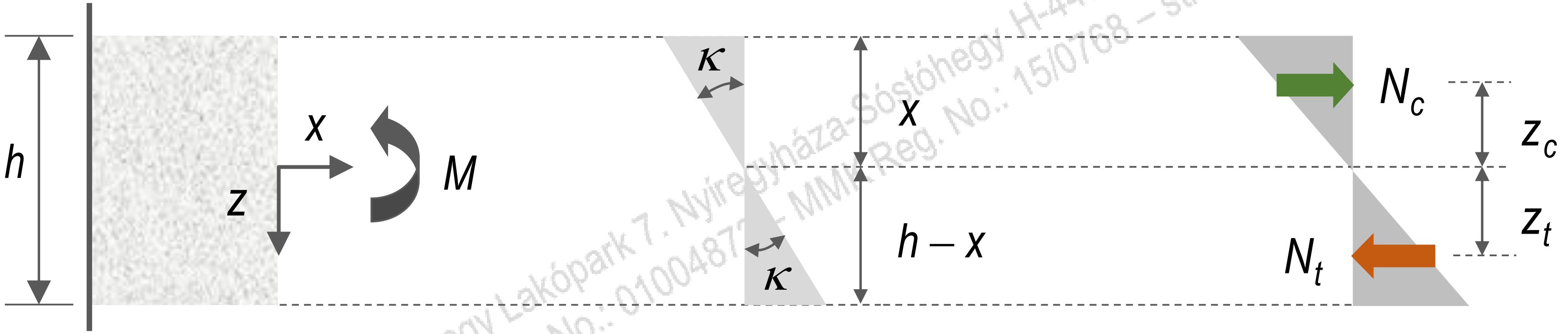
Arm of internal forces: $z_c = \frac{2}{3} \cdot x$

$z_t = \frac{2}{3} \cdot (h - x)$

Load process of concrete member in bending – elastic state

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

2. Moment equilibrium:

$$M = N_c \cdot z_c + N_t \cdot z_t$$

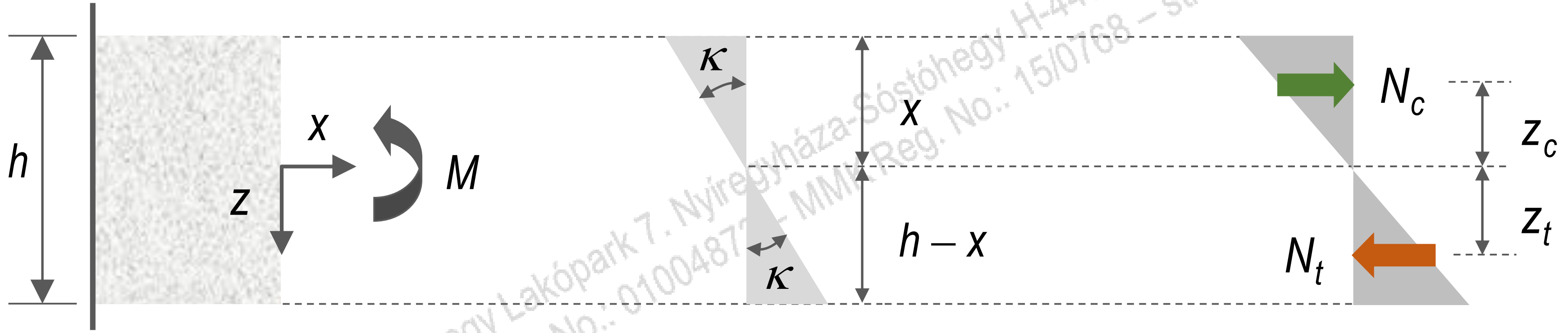
$$M = \frac{1}{2} \cdot (\kappa \cdot x \cdot E_{cd}) \cdot (x \cdot b) \cdot \frac{2}{3} \cdot x + \frac{1}{2} \cdot (\kappa \cdot [h - x] \cdot E_{cd}) \cdot ([h - x] \cdot b) \cdot \frac{2}{3} \cdot (h - x)$$

$$M = \kappa \cdot x \cdot b \cdot E_{cd} \cdot \frac{x^2}{3} + \kappa \cdot b \cdot (h - x) \cdot E_{cd} \frac{(h - x)^2}{3} = \kappa \cdot E_{cd} \cdot \frac{b \cdot h^3}{12} = \kappa \cdot E_{cd} \cdot I_x$$

Load process of concrete member in bending – elastic state

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



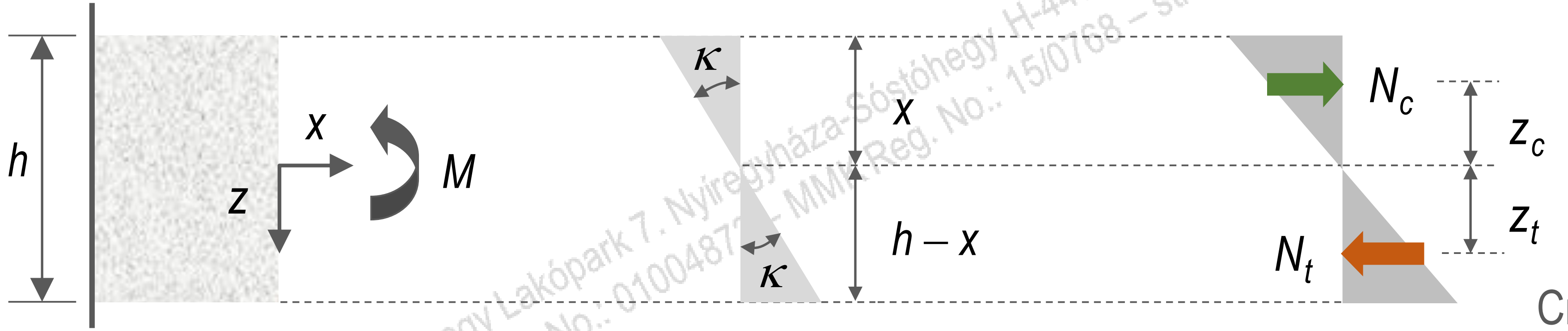
$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

$$\kappa = \frac{\varepsilon_t}{h - x} = \frac{\frac{\sigma_t}{E_{cd}}}{\frac{h}{2}} = \frac{2 \cdot \sigma_t}{h \cdot E_{cd}} \Rightarrow M = \kappa \cdot E_{cd} \cdot I = \frac{2 \cdot \sigma_t}{h \cdot E_{cd}} \cdot E_{cd} \cdot \frac{b \cdot h^3}{12} = \sigma_t \cdot \frac{b \cdot h^2}{6}$$

Load process of concrete member in bending – elastic state

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \frac{f_{ctd}}{E_{cd}} \qquad \sigma_t = f_{ctd}$$

Cracking in the most tensioned fibre of the concrete cross-section!

$$\kappa = \frac{\varepsilon_t}{h-x} = \frac{\frac{f_{ctd}}{E_{cd}}}{\frac{h}{2}} = \frac{2 \cdot f_{ctd}}{h \cdot E_{cd}}$$

$$M_{Rd} = \kappa \cdot E_{cd} \cdot I = \frac{2 \cdot f_{ctd}}{h \cdot E_{cd}} \cdot E_{cd} \cdot \frac{b \cdot h^3}{12} = f_{ctd} \cdot \frac{b \cdot h^2}{6}$$

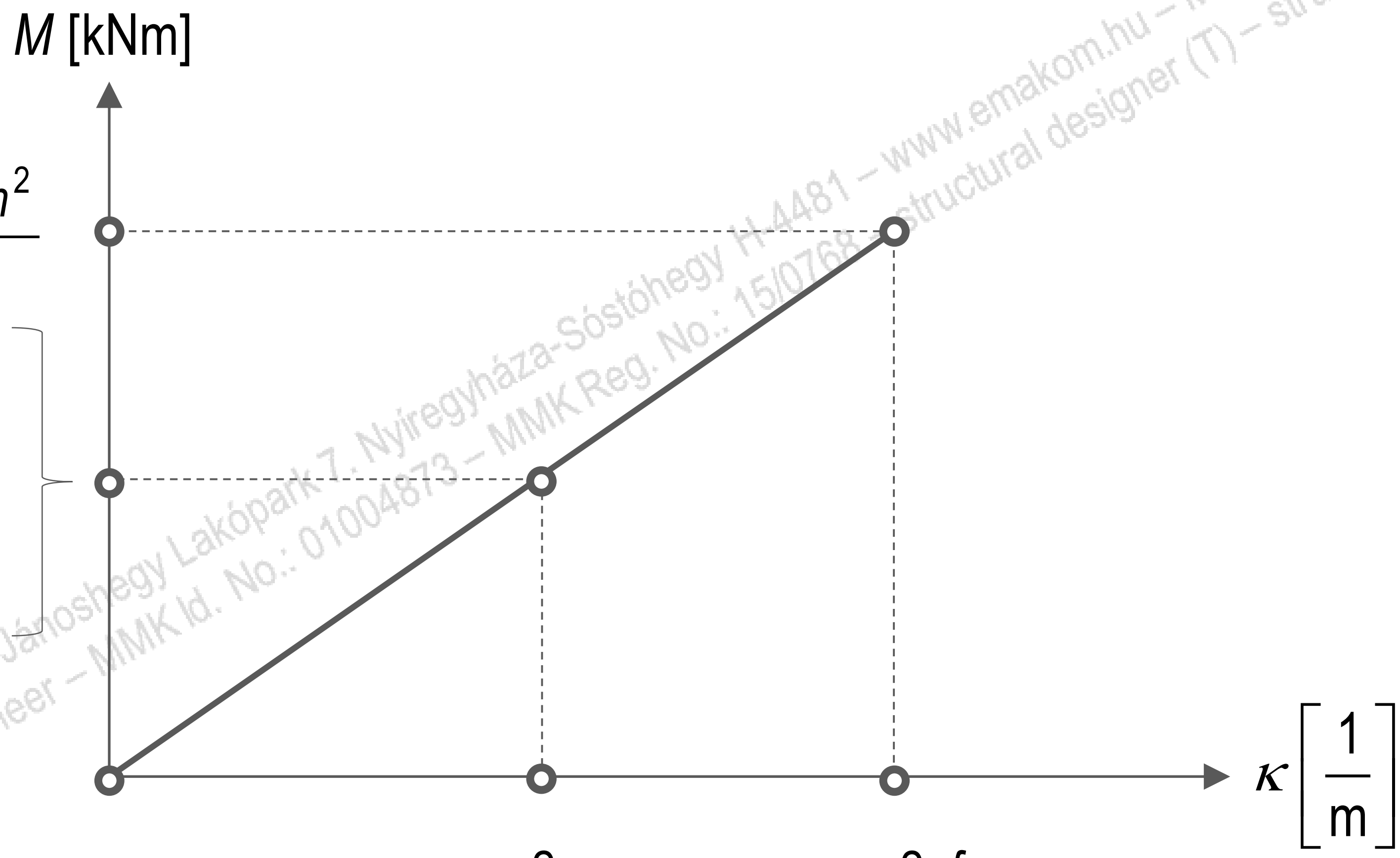
Moment resistance

Bending moment-Curvature ($\boldsymbol{M} - \boldsymbol{\kappa}$) relationship for concrete cross-section

$$M_{Rd} = f_{ctd} \cdot \frac{b \cdot h^2}{6}$$

$$M = \sigma_t \cdot \frac{b \cdot h^2}{6}$$

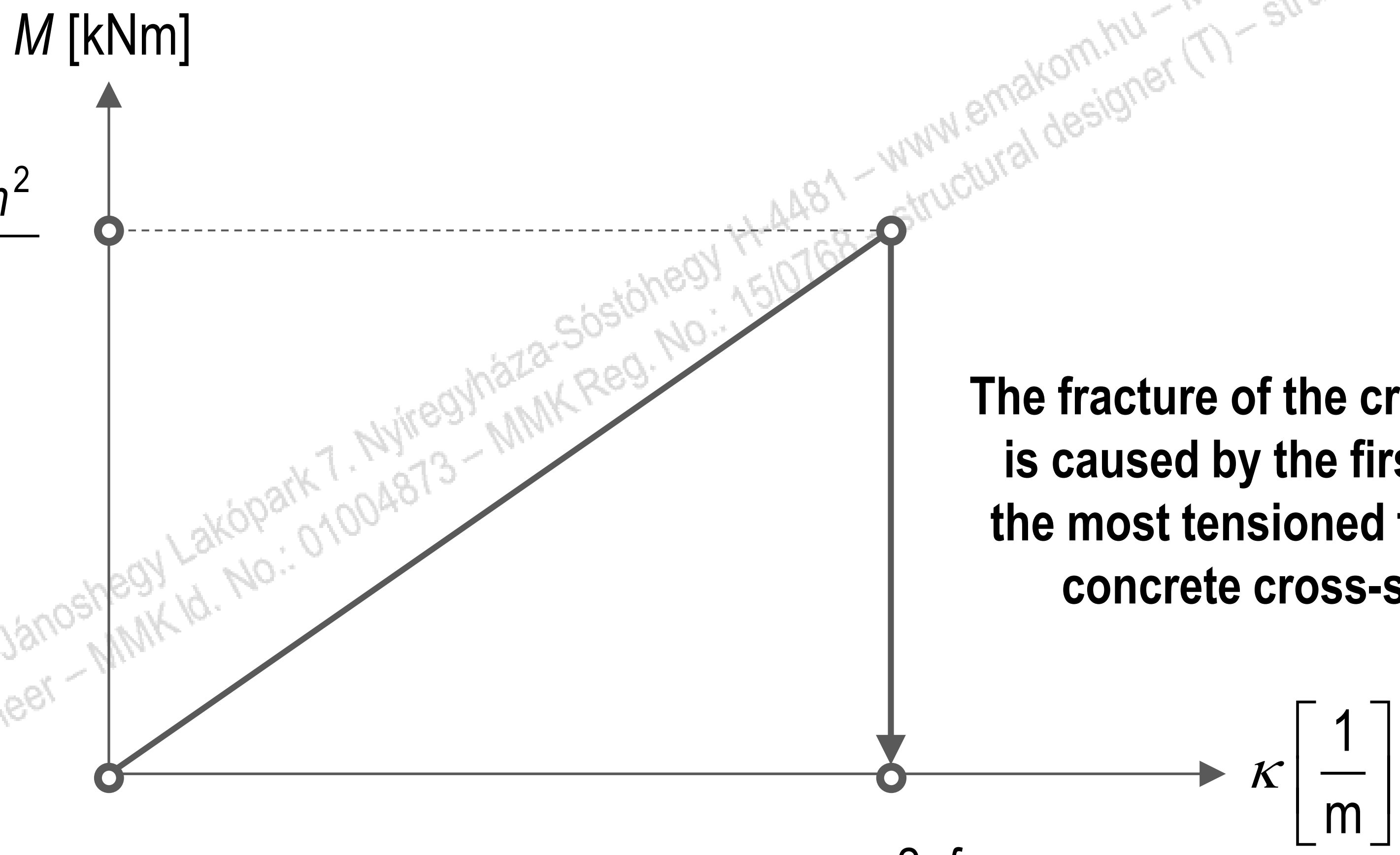
$$\sigma_t = \frac{6 \cdot M}{b \cdot h^2}$$



$$\kappa = \frac{2 \cdot \sigma_t}{h \cdot E_{cd}} \qquad \kappa_{Rd} = \frac{2 \cdot f_{ctd}}{h \cdot E_{cd}}$$

Bending moment-Curvature ($\boldsymbol{M} - \boldsymbol{\kappa}$) relationship for concrete cross-section

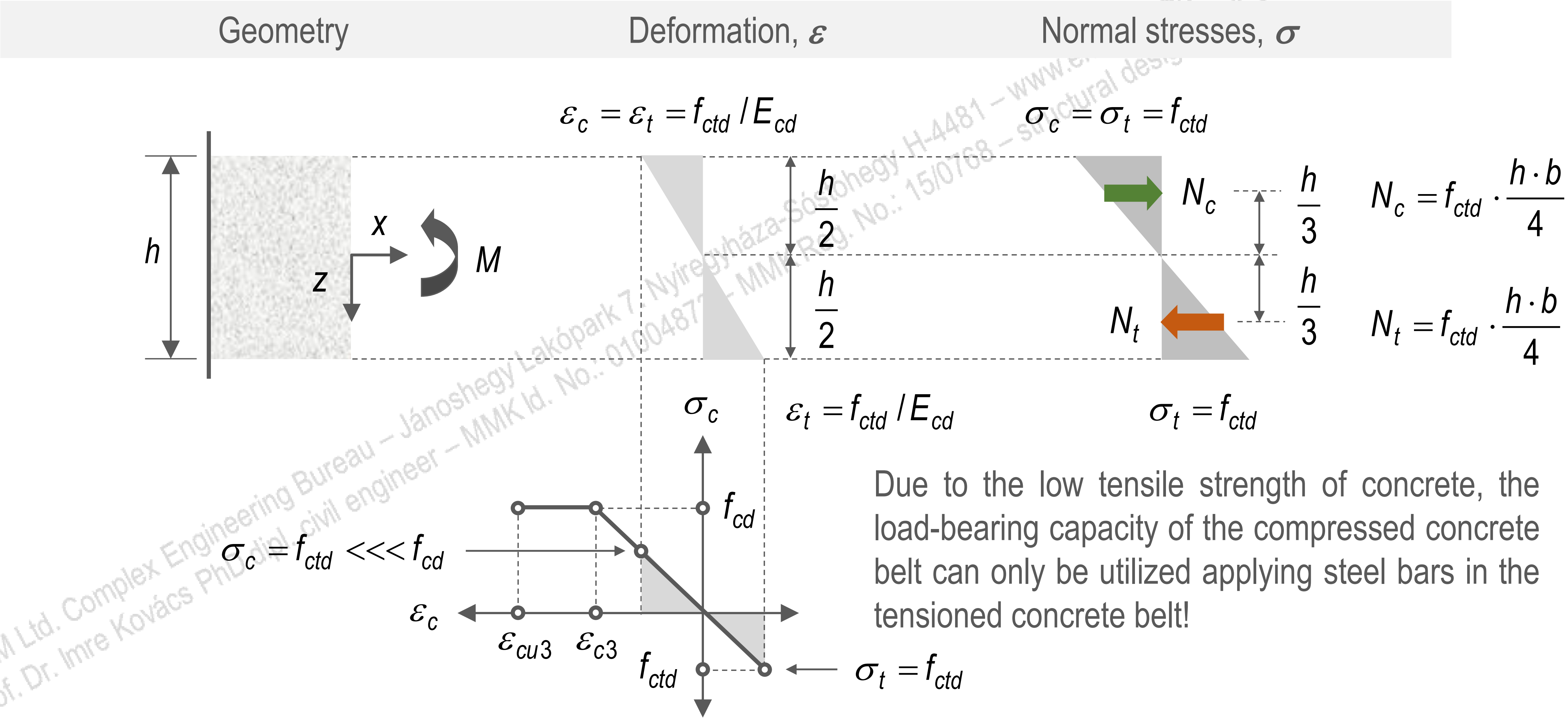
$$M_{Rd} = f_{ctd} \cdot \frac{b \cdot h^2}{6}$$



$$\kappa_{Rd} = \frac{2 \cdot f_{ctd}}{h \cdot E_{cd}}$$

The fracture of the cross-section is caused by the first crack in the most tensioned fibre of the concrete cross-section!

Load process of concrete member in bending – elastic state



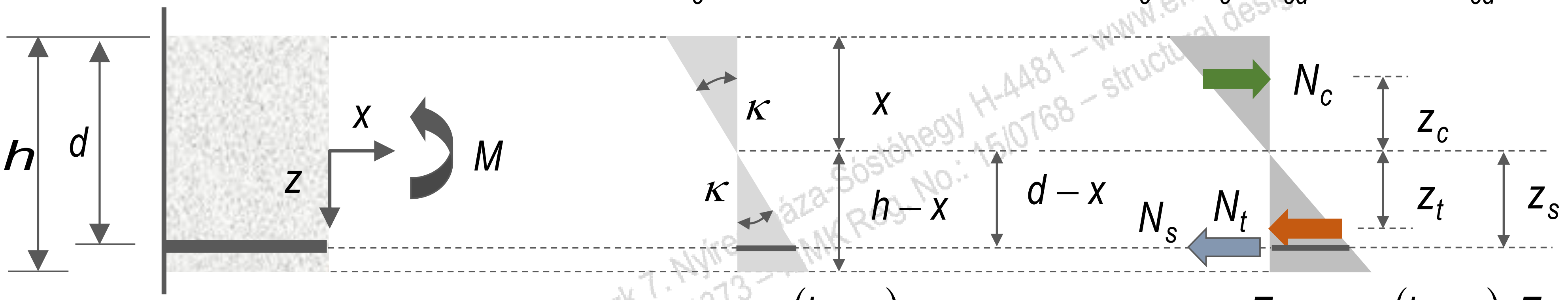
Due to the low tensile strength of concrete, the load-bearing capacity of the compressed concrete belt can only be utilized applying steel bars in the tensioned concrete belt!

Load process of reinforced concrete member in bending – elastic state (uncracked state)

Geometry	Deformation, ε	Normal stresses, σ

$\varepsilon_c = \kappa \cdot x$

$\sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$



$\varepsilon_t = \kappa \cdot (h - x)$

$\varepsilon_s = \kappa \cdot (d - x)$

$\sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$

$\sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$

1. Horizontal force equilibrium:

$\Sigma N = 0 \rightarrow 0 = N_c - N_t - N_s$

Internal forces:

$$N_c = \frac{1}{2} \cdot (\kappa \cdot x) \cdot E_{cd} \cdot x \cdot b$$

$$N_t = \frac{1}{2} \cdot \kappa \cdot (h - x) \cdot E_{cd} \cdot (h - x) \cdot b - \kappa \cdot (d - x) \cdot E_{cd} \cdot A_s$$

$$N_s = \kappa \cdot (d - x) \cdot E_s \cdot A_s$$

Arm of the internal forces:

$$z_c = \frac{2}{3} \cdot x$$

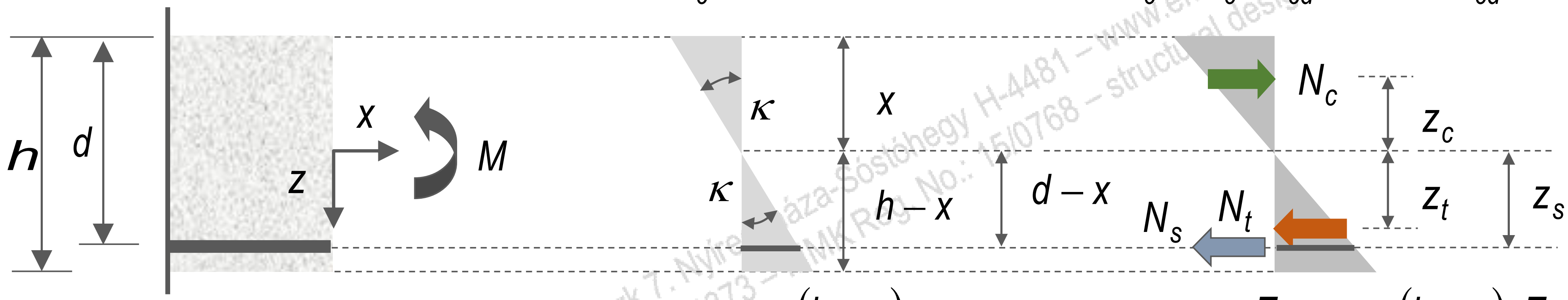
$$z_t = \frac{2}{3} \cdot (h - x)$$

$$z_s = (d - x)$$

Load process of reinforced concrete member in bending – elastic state (uncracked state)

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

$$\varepsilon_s = \kappa \cdot (d - x) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$$

1. Horizontal force equilibrium:

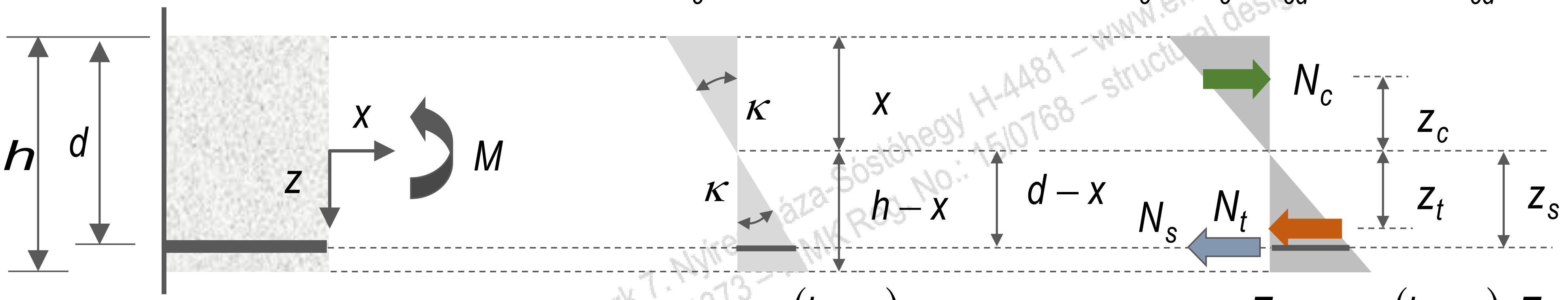
$$\Sigma N = 0 \quad \rightarrow \quad 0 = N_c - N_t - N_s$$

$$0 = \frac{1}{2} \cdot (\kappa \cdot x) \cdot E_{cd} \cdot x \cdot b - \left[\frac{1}{2} \cdot \kappa \cdot (h - x) \cdot E_{cd} \cdot (h - x) \cdot b - \kappa \cdot (d - x) \cdot E_{cd} \cdot A_s \right] - \kappa \cdot (d - x) \cdot E_s \cdot A_s$$

Load process of reinforced concrete member in bending – elastic state (uncracked state)

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

$$\varepsilon_s = \kappa \cdot (d - x) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$$

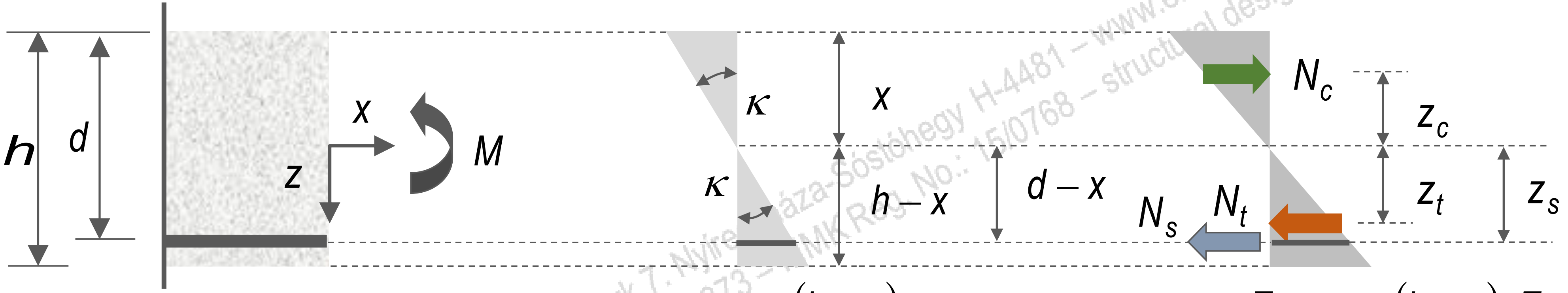
1. Horizontal force equilibrium: $\Sigma N = 0 \rightarrow 0 = N_c - N_t - N_s$ Static moment on the axis of bending

$$0 = \kappa \cdot \left\{ \frac{1}{2} \cdot b \cdot x^2 - \frac{1}{2} \cdot b \cdot (h - x)^2 - A_s \cdot (d - x) \cdot \left(\frac{E_s}{E_{cd}} - 1 \right) \right\} \Rightarrow \alpha = \frac{E_s}{E_{cd}} \Rightarrow 0 = \kappa \cdot \left\{ \frac{1}{2} \cdot b \cdot x^2 - \frac{1}{2} \cdot b \cdot (h - x)^2 - A_s \cdot (d - x) \cdot (\alpha - 1) \right\}$$

Load process of reinforced concrete member in bending – elastic state (uncracked state)

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x) \cdot E_{cd}$$

$$\varepsilon_s = \kappa \cdot (d - x) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x) \cdot E_s$$

1. Horizontal force equilibrium:

$$\Sigma N = 0 \quad \rightarrow \quad 0 = N_c - N_t - N_s$$

Neutral axis / Depth of the compressed belt

$$x = x_I = \frac{\frac{1}{2} \cdot b \cdot h^2 + A_s \cdot (\alpha - 1) \cdot d}{b \cdot h + A_s \cdot (\alpha - 1)} = \frac{S_I}{A_I}$$

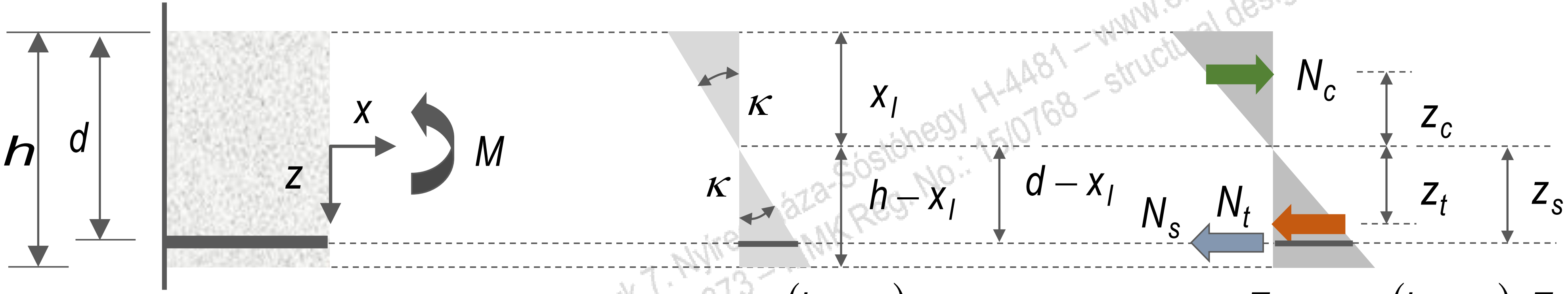
Static moment on the most compressed fibre of the cross-section

Ideal cross-sectional area in the elastic (uncracked) state

Load process of reinforced concrete member in bending – elastic state (uncracked state)

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x_I \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x_I \cdot E_{cd}$$



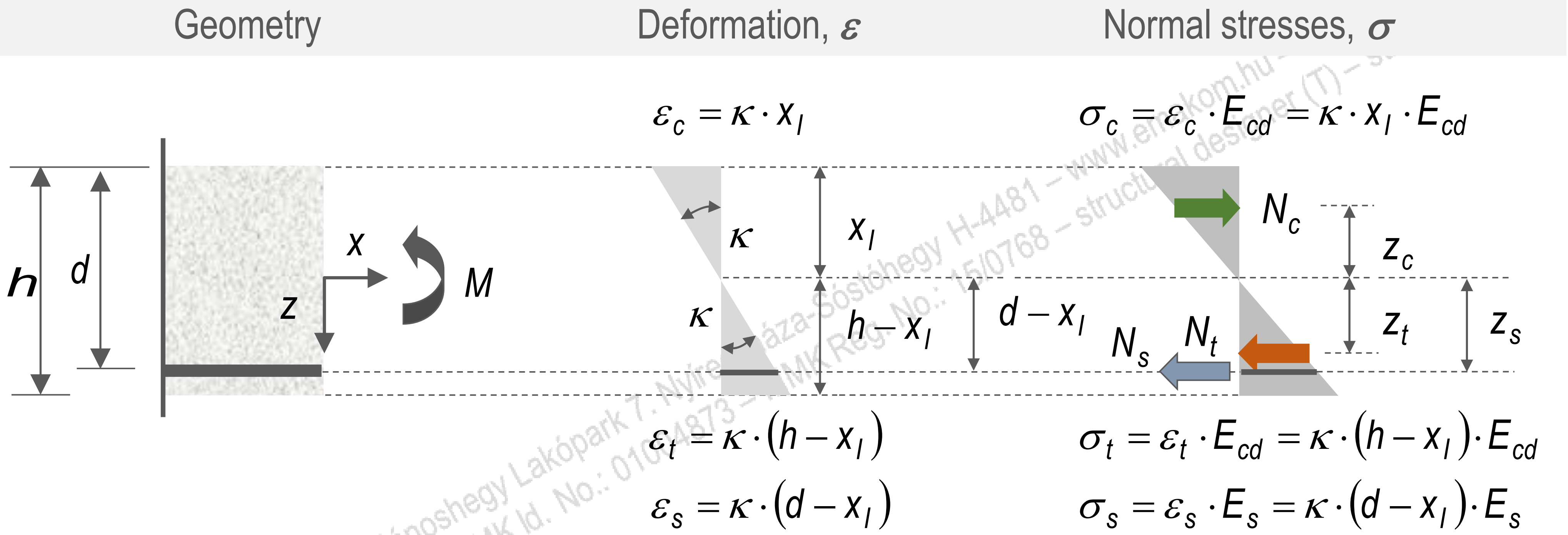
$$\varepsilon_t = \kappa \cdot (h - x_I) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x_I) \cdot E_{cd}$$

$$\varepsilon_s = \kappa \cdot (d - x_I) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x_I) \cdot E_s$$

2. Bending moment equilibrium: $\Sigma M = 0 \rightarrow M = N_c \cdot z_c + N_t \cdot z_t + N_s \cdot z_s$

$$M = \left\{\frac{1}{2}\cdot(\kappa \cdot x_I)\cdot E_{cd}\cdot b \cdot x_I\right\}\cdot\frac{2}{3}\cdot x_I + \left\{\left\{\frac{1}{2}\cdot\kappa\cdot(h-x_I)\cdot E_{cd}\cdot b\cdot(h-x_I)\right\}\cdot\frac{2}{3}\cdot(h-x_I) - \kappa\cdot(d-x_I)\cdot E_{cd}\cdot A_s\cdot(d-x_I)\right\} + \kappa\cdot(d-x_I)\cdot E_s\cdot A_s\cdot(d-x_I)$$

Load process of reinforced concrete member in bending – elastic state (uncracked state)



2. Bending moment equilibrium:

$$\Sigma M = 0 \quad \rightarrow \quad M = N_c \cdot z_c + N_t \cdot z_t + N_s \cdot z_s$$

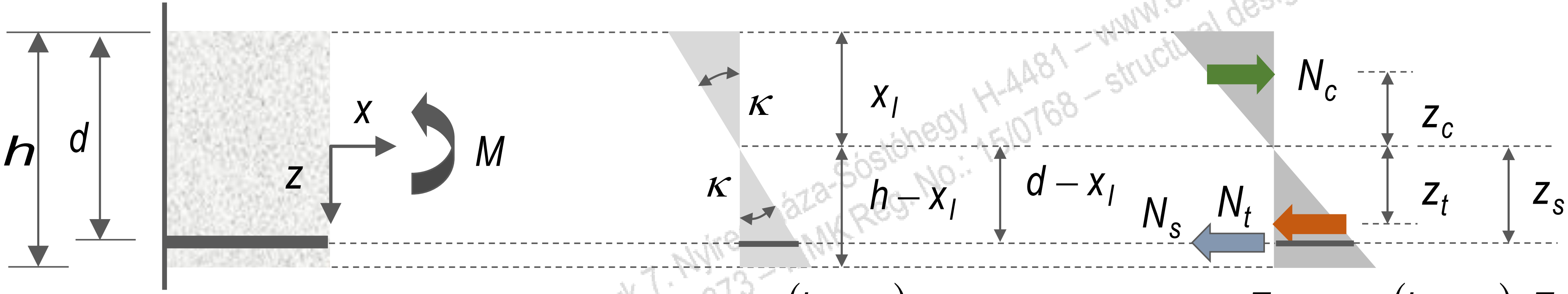
Moment of inertia of the elastic (uncracked) cross-section

$$M = \kappa \cdot E_{cd} \cdot \left\{ \frac{b \cdot x_I^3}{3} + \frac{(h - x_I)^3 \cdot b}{3} + A_s \cdot (\alpha - 1) \cdot (d - x_I)^2 \right\} \Rightarrow M = \kappa \cdot E_{cd} \cdot I_I$$

Load process of reinforced concrete member in bending – elastic state (uncracked state)

Geometry	Deformation, ε	Normal stresses, σ

$$\varepsilon_c = \kappa \cdot x_I \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa \cdot x_I \cdot E_{cd}$$



$$\varepsilon_t = \kappa \cdot (h - x_I) \qquad \sigma_t = \varepsilon_t \cdot E_{cd} = \kappa \cdot (h - x_I) \cdot E_{cd}$$

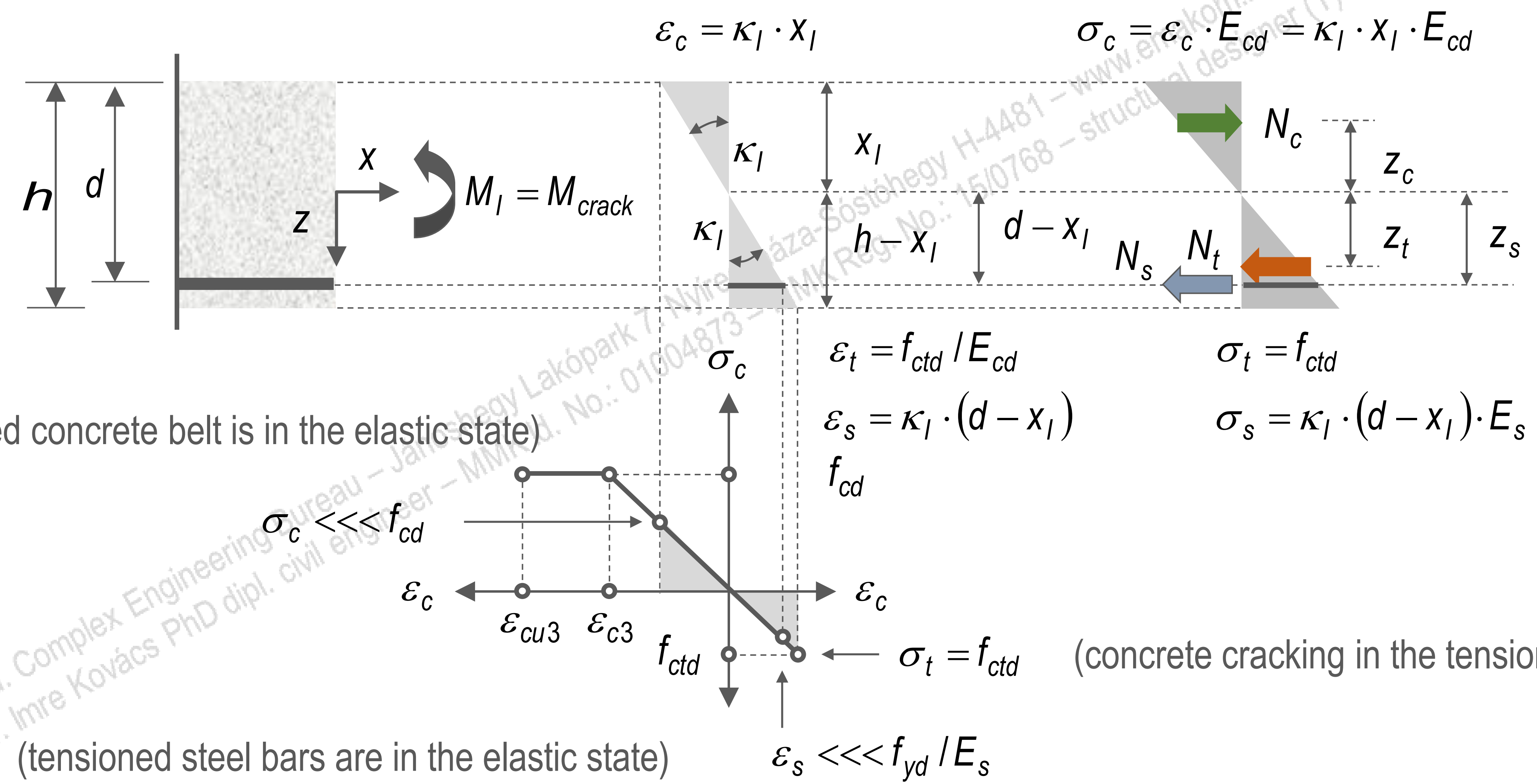
$$\varepsilon_s = \kappa \cdot (d - x_I) \qquad \sigma_s = \varepsilon_s \cdot E_s = \kappa \cdot (d - x_I) \cdot E_s$$

Cross-sectional moduli on the most tensioned fibre of the cross-section

$$\kappa = \frac{\varepsilon_t}{h - x_I} = \frac{\sigma_t}{E_{cd}} \cdot \frac{1}{h - x_I} \Rightarrow M = \kappa \cdot E_{cd} \cdot I_I = \sigma_t \cdot \frac{\left\{ \frac{b \cdot x_I^3}{3} + \frac{(h - x_I)^3 \cdot b}{3} + A_s \cdot (\alpha - 1) \cdot (d - x_I)^2 \right\}}{h - x_I} = \sigma_t \cdot \frac{I_I}{h - x_I} = \sigma_t \cdot W_I^t$$

Cracking at the most tensioned fibre of the cross-section

Geometry | Deformation, ε | Normal stresses, σ



(compressed concrete belt is in the elastic state)

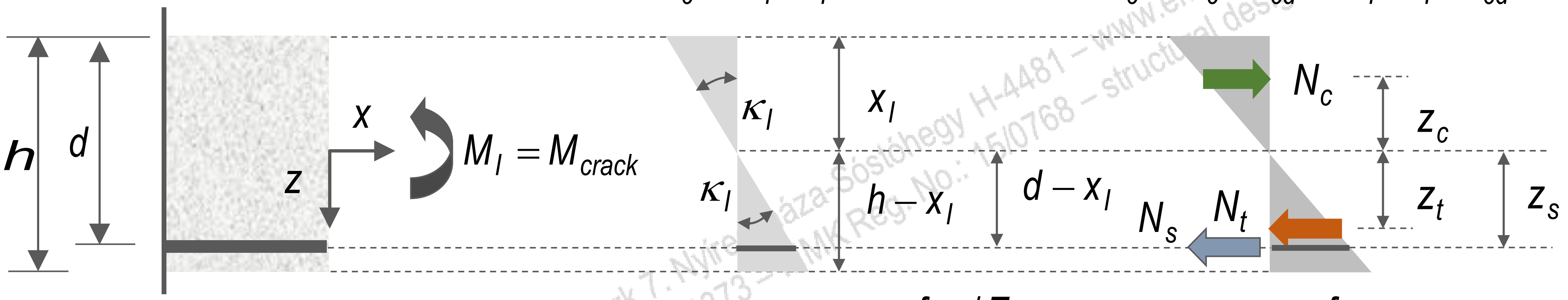
(concrete cracking in the tensioned belt)

(tensioned steel bars are in the elastic state)

Cracking at the most tensioned fibre of the cross-section – M_{crack}

Geometry | Deformation, ε | Normal stresses, σ

$$\varepsilon_c = \kappa_I \cdot x_I \qquad \sigma_c = \varepsilon_c \cdot E_{cd} = \kappa_I \cdot x_I \cdot E_{cd}$$



$$\varepsilon_t = f_{ctd} / E_{cd} \qquad \sigma_t = f_{ctd}$$

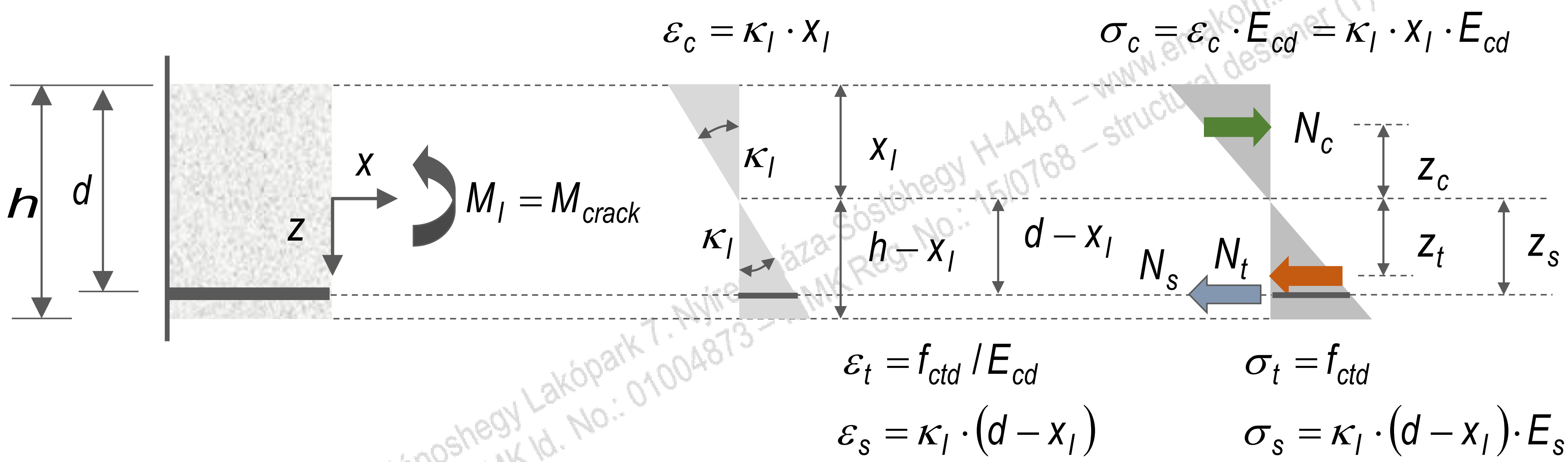
$$\varepsilon_s = \kappa_I \cdot (d - x_I) \qquad \sigma_s = \kappa_I \cdot (d - x_I) \cdot E_s$$

Cracking moment of the RC cross-section

$$\kappa_I = \frac{\varepsilon_t}{h - x_I} = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h - x_I} \Rightarrow M_{crack} = \kappa_I \cdot E_{cd} \cdot I_I = f_{ctd} \cdot \frac{\left\{ \frac{b \cdot x_I^3}{3} + \frac{(h - x_I)^3 \cdot b}{3} + A_s \cdot (\alpha - 1) \cdot (d - x_I)^2 \right\}}{h - x_I} = f_{ctd} \cdot \frac{I_I}{h - x_I} = f_{ctd} \cdot W_I^t$$

Cracking at the most tensioned fibre of the cross-section – $\sigma_{c,I} - \sigma_{s,I}$

Geometry	Deformation, ε	Normal stresses, σ



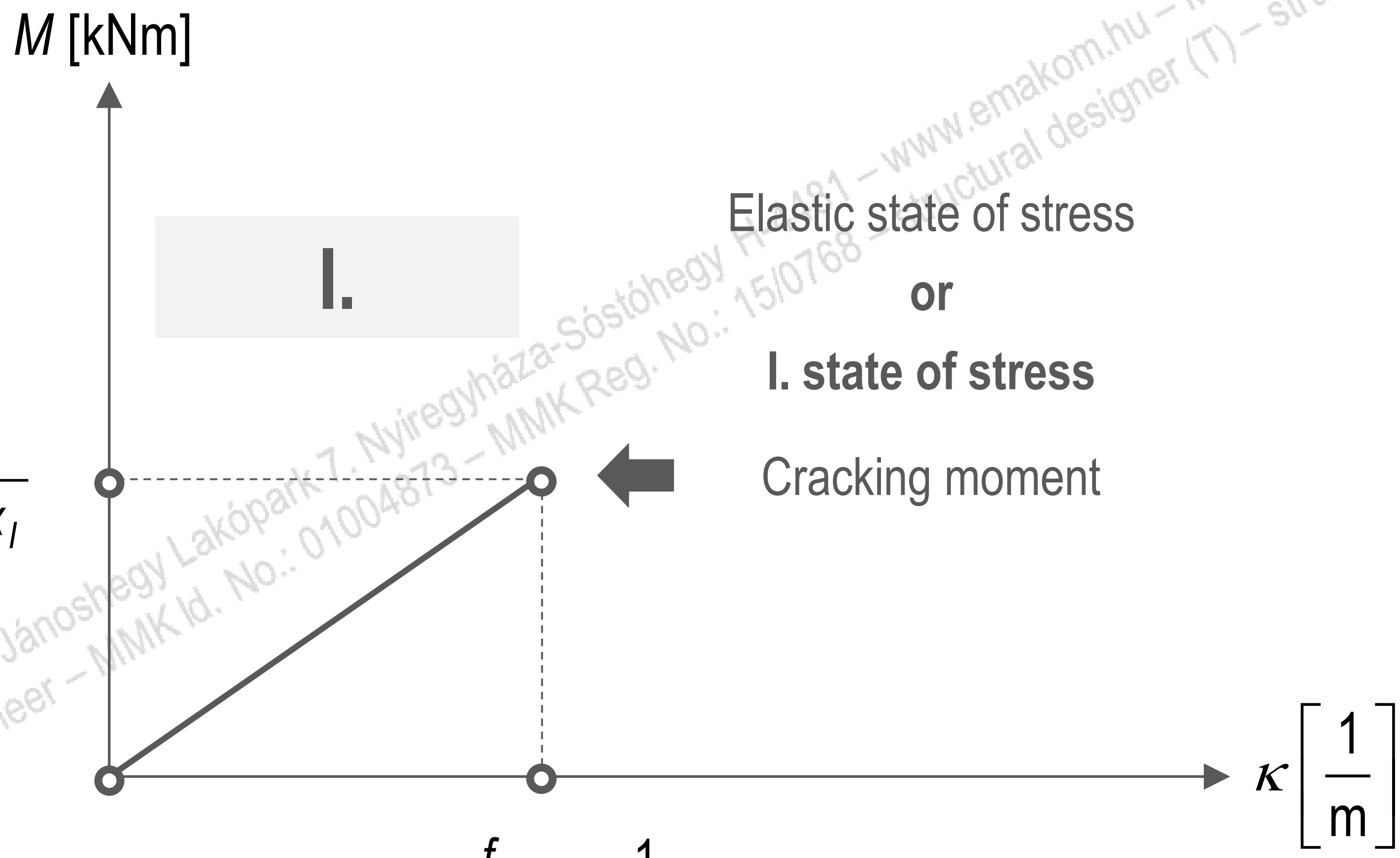
$$\sigma_{c,I} = \varepsilon_{c,I} \cdot E_{cd} = \kappa_I \cdot x_I \cdot E_{cd} \Rightarrow \sigma_{c,I} = \frac{M_{crack}}{I_I} \cdot x_I$$

Compressive stress of concrete in the most compressed fibre of the cross-section at the moment of concrete cracking

$$\sigma_{s,I} = \varepsilon_{s,I} \cdot E_s = \kappa_I \cdot (d - x_I) \cdot E_s \Rightarrow \sigma_{s,I} = \alpha \cdot \frac{M_{crack}}{I_I} \cdot (d - x_I)$$

Tensile stress of reinforcement at the moment of concrete cracking

Bending moment-Curvature ($M-\kappa$) relationship for RC cross-section - Cracking



$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h-x_I}$$

$$\kappa_I = \frac{f_{ctd}}{E_{cd}} \cdot \frac{1}{h-x_I}$$

Elastic state of stress

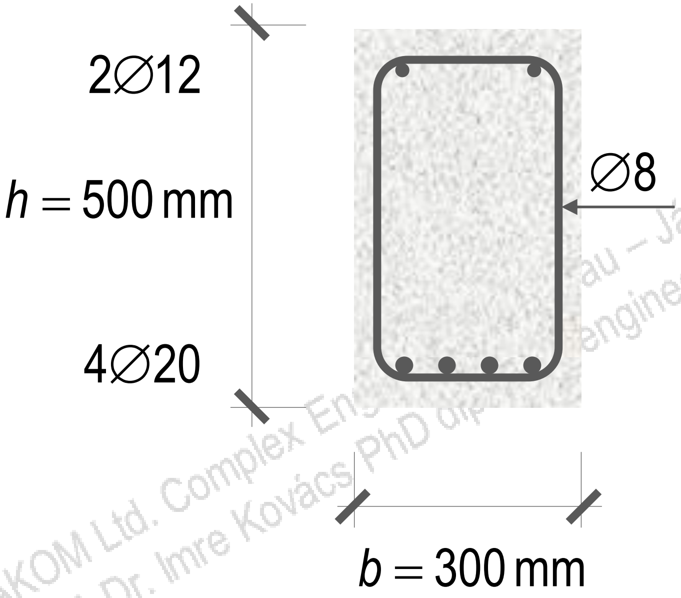
or

I. state of stress

Cracking moment

Example: Analysis of the RC cross-section in the uncracked state of stress (1)

Determine the cracking moment of the outlined rectangular RC cross-section (**h = 500 mm, b = 300 mm**) in the persistent and transient design situation**,** if the concrete grade is **C30/37**, the nominal concrete cover on stirrups is **C_{nom} = 30 mm**, the maximum size of the aggregates is **d_g = 16 mm**, the diamater of the stirrup reinforcement is **$Ø_s$ = 8 mm**, the tensioned reinforcement consists of **4Ø20,** the auxalary (compressed side) reinforcement consists of **2Ø12, B500A**! Determine and outline the strain and stress distribution of the cross-section at the moment of concrete cracking!



- Concrete grade: **C30/37**
- Aggregate: **d_g = 16 mm**
- RE bar grade: **B500A**
- Width of cross-section: **b = 300 mm**
- Height of cross-section: **h = 500 mm**
- Concrete cover: **C_{nom} = 30 mm**
- Tensioned reinforcement: **$A_{s,prov}$ = 4Ø20 (1256 mm²)**
- Auxalary reinforcement: **$A_{s}'_{,prov}$ = 2Ø12**
- Stirrups: **Ø8**

Example: Analysis of the RC cross-section in the uncracked state of stress (1)

- $f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{30}{1{,}50} = 20\,\text{N/mm}^2$ ☑
  - $f_{ctm} = 0{,}30 \cdot f_{ck}^{(2/3)} = 0{,}30 \cdot 30^{(2/3)} = 2{,}90\,\text{N/mm}^2$ ☑
  - $f_{ctk0{,}05} = 0{,}70 \cdot f_{ctm} = 0{,}70 \cdot 2{,}90 = 2{,}00\,\text{N/mm}^2$ ☑
  - $f_{ctk,fl0{,}05} = \max\left\{ \begin{array}{c} \left(1{,}6 - \frac{h}{1000}\right) \cdot f_{ctk0{,}05} \\ f_{ctk0{,}05} \end{array} \right\} = \max\left\{ \begin{array}{c} \left(1{,}6 - \frac{500}{1000}\right) \cdot 2{,}00 = 2{,}20\,\text{N/mm}^2 \\ 2{,}00\,\text{N/mm}^2 \end{array} \right\} = 2{,}20\,\text{N/mm}^2$ ☑
- $f_{ctd} = \alpha_{ct} \cdot \frac{f_{ctk,fl0{,}05}}{\gamma_C} = 1{,}00 \cdot \frac{2{,}20}{1{,}50} = 1{,}47\,\text{N/mm}^2$ ☑
- $E_{cd} = \frac{f_{cd}}{1{,}75\,‰} = \frac{20}{1{,}75} \approx 11400\,\text{N/mm}^2$ ☑
- $f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}15} = 435\,\text{N/mm}^2$ ☑
- $\alpha = E_s / E_{cd} = 200000/11400 \approx 17{,}5$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (1)

- $$\Delta\varnothing_{min} = \max\begin{Bmatrix} k_1 \cdot \varnothing = 1 \cdot 20 = 20\,\text{mm} \\ d_g + k_2 = 16 + 5 = 21\,\text{mm} \\ 20\,\text{mm} \end{Bmatrix} = 21\,\text{mm}$$ ☑

- $$\Delta\varnothing = \frac{b - (2 \cdot C_{nom} + 2 \cdot \varnothing_s + n \cdot \varnothing)}{n-1} = \frac{300 - (2 \cdot 30 + 2 \cdot 8 + 4 \cdot 20)}{4-1} = 48\,\text{mm} > \Delta\varnothing_{min} = 21\,\text{mm}$$ ☑

- $$d = h - \left(C_{nom} + \varnothing_s + \frac{\varnothing}{2}\right) = 500 - \left(30 + 8 + \frac{20}{2}\right) = 442\,\text{mm}$$ ☑

➡ $$A_I = b \cdot h + (\alpha - 1) \cdot A_{s,prov} = 300 \cdot 500 + (17{,}5 - 1) \cdot 4 \cdot 314 = 0{,}170 \cdot 10^6\ \text{mm}^2$$ ☑

- $$S_x' = b \cdot h \cdot \frac{h}{2} + (\alpha - 1) \cdot A_{s,prov} \cdot d = 300 \cdot 500 \cdot \frac{500}{2} + (17{,}5 - 1) \cdot 1256 \cdot 442 = 46{,}70 \cdot 10^6\ \text{mm}^3$$ ☑

➡ $$x_I = \frac{S_x'}{A_I} = \frac{46{,}70 \cdot 10^6}{0{,}17 \cdot 10^6} = 275\,\text{mm}$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (1)

$$I_I = \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_I\right)^2 + (\alpha - 1) \cdot A_{s,prov} \cdot (d - x_I)^2 =$$

$$= \frac{300 \cdot 500^3}{12} + 300 \cdot 500 \cdot \left(\frac{500}{2} - 275\right)^2 + (17{,}5 - 1) \cdot 1256 \cdot (442 - 275)^2 = 3{,}80 \cdot 10^9 \text{ mm}^4$$ ☑

$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I} = 1{,}47 \cdot \frac{3{,}80 \cdot 10^9}{500 - 275} = 24{,}80 \text{ kNm}$$ ☑

$$\sigma_{c,I} = \frac{M_{crack}}{I_I} \cdot x_I = \frac{24{,}80 \cdot 10^6}{3{,}80 \cdot 10^9} \cdot 275 = 1{,}79 \text{ N/mm}^2$$ ☑

$$\varepsilon_{c,I} = \frac{\sigma_{c,I}}{E_{cd}} = \frac{1{,}79}{11400} = 0{,}157 ‰$$ ☑

$$\sigma_{s,I} = \alpha \cdot \frac{M_{crack}}{I_I} \cdot (d - x_I) = 17{,}50 \cdot \frac{24{,}80 \cdot 10^6}{3{,}80 \cdot 10^9} \cdot (442 - 275) = 19{,}10 \text{ N/mm}^2$$ ☑

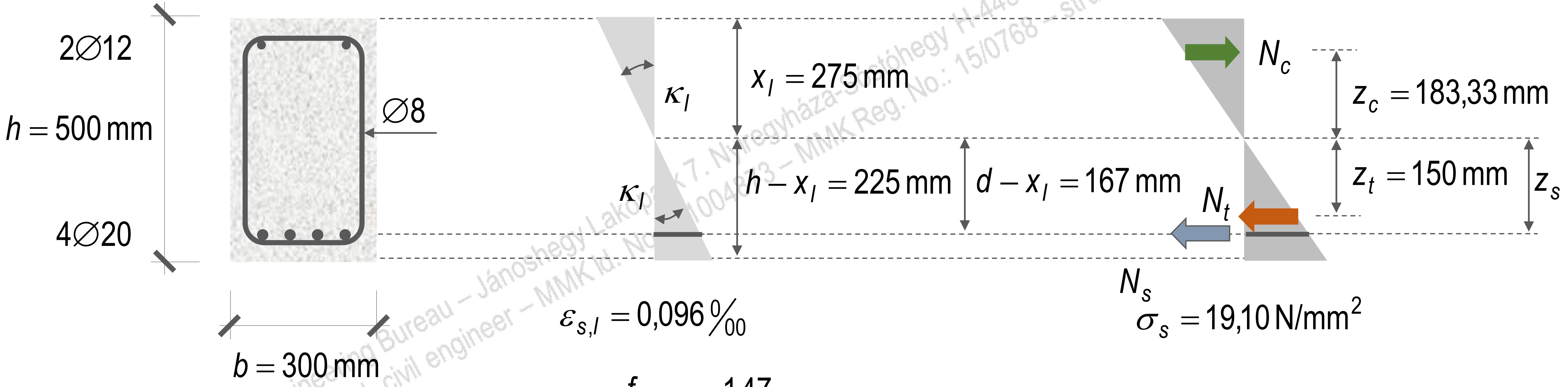
$$\varepsilon_{s,I} = \frac{\sigma_{s,I}}{E_s} = \frac{19{,}10}{200000} = 0{,}096 ‰$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (1)

Geometry	Deformation, ε	Normal stress, σ

$\varepsilon_{c,I} = 0{,}157\,‰$

$\sigma_c = 1{,}79\ \mathrm{N/mm^2}$



$\varepsilon_{s,I} = 0{,}096\,‰$

$\sigma_s = 19{,}10\ \mathrm{N/mm^2}$

$$\kappa_I = \frac{\varepsilon_{c,I}}{x_I} = \frac{0{,}157}{275} = 0{,}57 \cdot 10^{-6}\ \mathrm{1/mm}$$

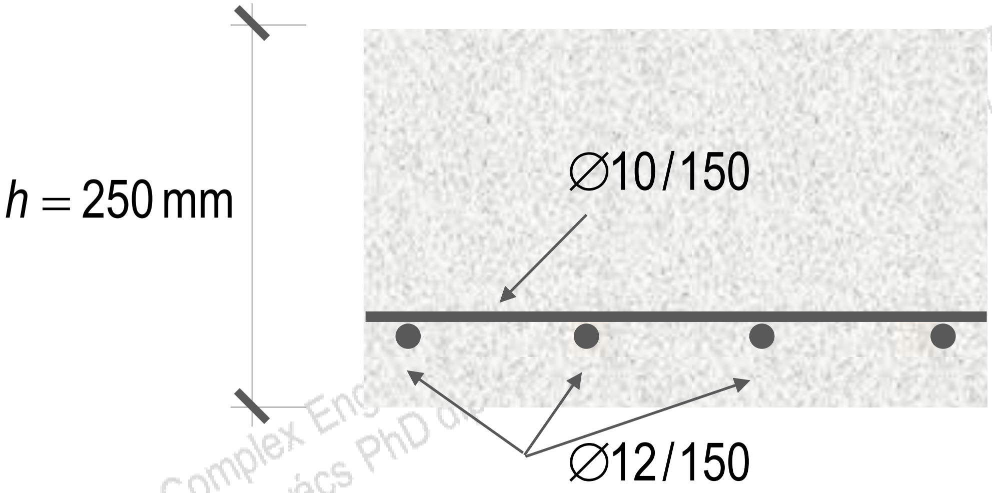
$$\varepsilon_t = \frac{f_{ctd}}{E_{cd}} = \frac{1{,}47}{11400} = 0{,}129\,‰$$

$$\sigma_t = f_{ctd} = 1{,}47\ \mathrm{N/mm^2}$$

$$M_I = M_{crack} = 24{,}80\ \mathrm{kNm}$$

Example: Analysis of the RC cross-section in the uncracked state of stress (2)

Determine the cracking moment of the outlined RC slab cross-section (***h* = 250 mm**), in the persistent and transient design situation, if the concrete grade is **C25/30**, the nominal concrete cover is **C_{nom} = 25 mm**, the maximum size of the aggregates is **d_g = 24 mm**, the main reinforcement consists of **Ø12/150,** the transversal reinforcement consists of **Ø10/150, B500B**! Determine and outline the strain and stress distribution of the cross-section at the moment of concrete cracking!



- Concrete grade: **C25/30**
- Aggregate: **d_g = 24 mm**
- RE bar grade: **B500B**
- Height of slab: ***h* = 250 mm**
- Concrete cover: **C_{nom} = 25 mm**
- Main reinforcement: **$a_{s,prov}$ = Ø12/150 (754 mm^2)**
- Transversal reinforcement: **$a_{s,prov,trans}$ = Ø10/150**

Example: Analysis of the RC cross-section in the uncracked state of stress (2)

- $f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{25}{1{,}50} = 16{,}66\,\text{N/mm}^2$ ☑
  - $f_{ctm} = 0{,}30 \cdot f_{ck}^{(2/3)} = 0{,}30 \cdot 25^{(2/3)} = 2{,}56\,\text{N/mm}^2$ ☑
  - $f_{ctk0{,}05} = 0{,}70 \cdot f_{ctm} = 0{,}70 \cdot 2{,}56 = 1{,}79\,\text{N/mm}^2$ ☑
  - $f_{ctk,fl0{,}05} = \max\left\{\begin{matrix}\left(1{,}6 - \frac{h}{1000}\right) \cdot f_{ctk0{,}05} \\ f_{ctk0{,}05}\end{matrix}\right\} = \max\left\{\begin{matrix}\left(1{,}6 - \frac{250}{1000}\right) \cdot 1{,}79 = 2{,}42\,\text{N/mm}^2 \\ 1{,}79\,\text{N/mm}^2\end{matrix}\right\} = 2{,}42\,\text{N/mm}^2$ ☑
- $f_{ctd} = \alpha_{ct} \cdot \frac{f_{ctk,fl0{,}05}}{\gamma_C} = 1{,}00 \cdot \frac{2{,}42}{1{,}50} = 1{,}61\,\text{N/mm}^2$ ☑
- $E_{cd} = \frac{f_{cd}}{1{,}75\,‰} = \frac{16{,}66}{1{,}75} \approx 9500\,\text{N/mm}^2$ ☑
- $f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}15} = 435\,\text{N/mm}^2$ ☑
- $\alpha = E_s / E_{cd} = 200000 / 9500 \approx 21$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (2)

- $d = h - \left(C_{nom} + \frac{\varnothing}{2}\right) = 250 - \left(25 + \frac{12}{2}\right) = 219\,\text{mm}$ ☑

➡ $a_I = b \cdot h + (\alpha - 1) \cdot a_{s,prov} = 1000 \cdot 250 + (21 - 1) \cdot 754 = 0{,}265 \cdot 10^6\ \text{mm}^2/\text{m}$ ☑

- $s_x^{'} = b \cdot h \cdot \frac{h}{2} + (\alpha - 1) \cdot a_{s,prov} \cdot d = 1000 \cdot 250 \cdot \frac{250}{2} + (21 - 1) \cdot 754 \cdot 219 = 34{,}55 \cdot 10^6\ \text{mm}^3/\text{m}$ ☑

➡ $x_I = \frac{s_x^{'}}{a_I} = \frac{34{,}55 \cdot 10^6}{0{,}265 \cdot 10^6} = 130\,\text{mm}$ ☑

➡ $i_I = \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_I\right)^2 + (\alpha - 1) \cdot a_{s,prov} \cdot (d - x_I)^2 =$

$= \frac{1000 \cdot 250^3}{12} + 1000 \cdot 250 \cdot \left(\frac{250}{2} - 130\right)^2 + (21 - 1) \cdot 754 \cdot (219 - 130)^2 = 1{,}43 \cdot 10^9\ \text{mm}^4/\text{m}$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (2)

$$m_I = m_{crack} = f_{ctd} \cdot \frac{i_I}{h - x_I} = 1{,}61 \cdot \frac{1{,}43 \cdot 10^9}{250 - 130} = 7{,}40 \text{ kNm/m}$$ ☑

$$\sigma_{c,I} = \frac{m_{crack}}{i_I} \cdot x_I = \frac{7{,}40 \cdot 10^6}{1{,}43 \cdot 10^9} \cdot 130 = 0{,}67 \text{ N/mm}^2$$ ☑

$$\varepsilon_{c,I} = \frac{\sigma_{c,I}}{E_{cd}} = \frac{0{,}67}{9500} = 0{,}071 ‰$$ ☑

$$\sigma_{s,I} = \alpha \cdot \frac{m_{crack}}{i_I} \cdot (d - x_I) = 21 \cdot \frac{7{,}40 \cdot 10^6}{1{,}43 \cdot 10^9} \cdot (219 - 130) = 9{,}67 \text{ N/mm}^2$$ ☑

$$\varepsilon_{s,I} = \frac{\sigma_{s,I}}{E_s} = \frac{9{,}67}{200000} = 0{,}048 ‰$$ ☑

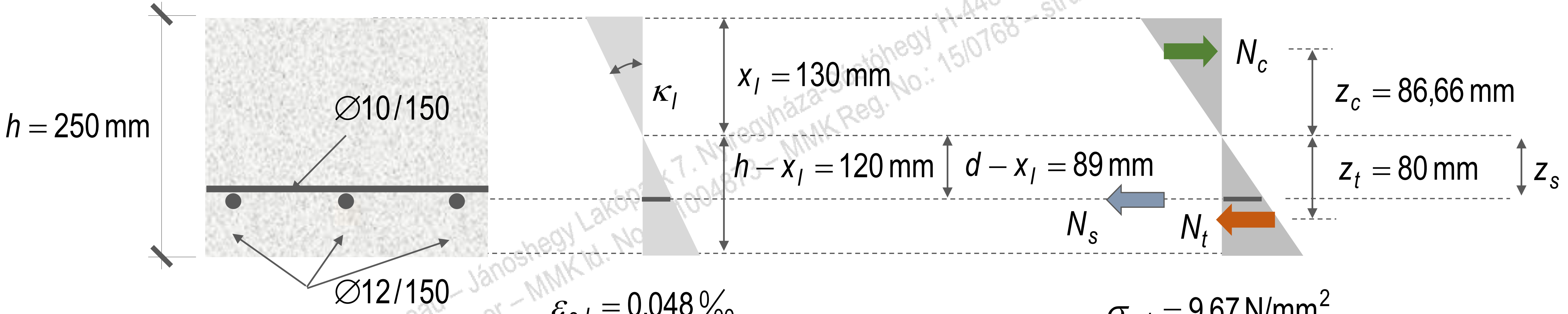
$$\kappa_I = \frac{\varepsilon_{c,I}}{x_I} = \frac{0{,}071}{130} = 0{,}546 \cdot 10^{-6} \ 1/\text{mm}$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (2)

Geometry	Deformation, ε	Normal stress, σ

$\varepsilon_{c,I} = 0{,}071\,‰$

$\sigma_{c,I} = 0{,}67\ \text{N/mm}^2$



$\varepsilon_{s,I} = 0{,}048\,‰$

$\sigma_{s,I} = 9{,}67\ \text{N/mm}^2$

$m_I = m_{crack} = 7{,}40\ \text{kNm/m}$

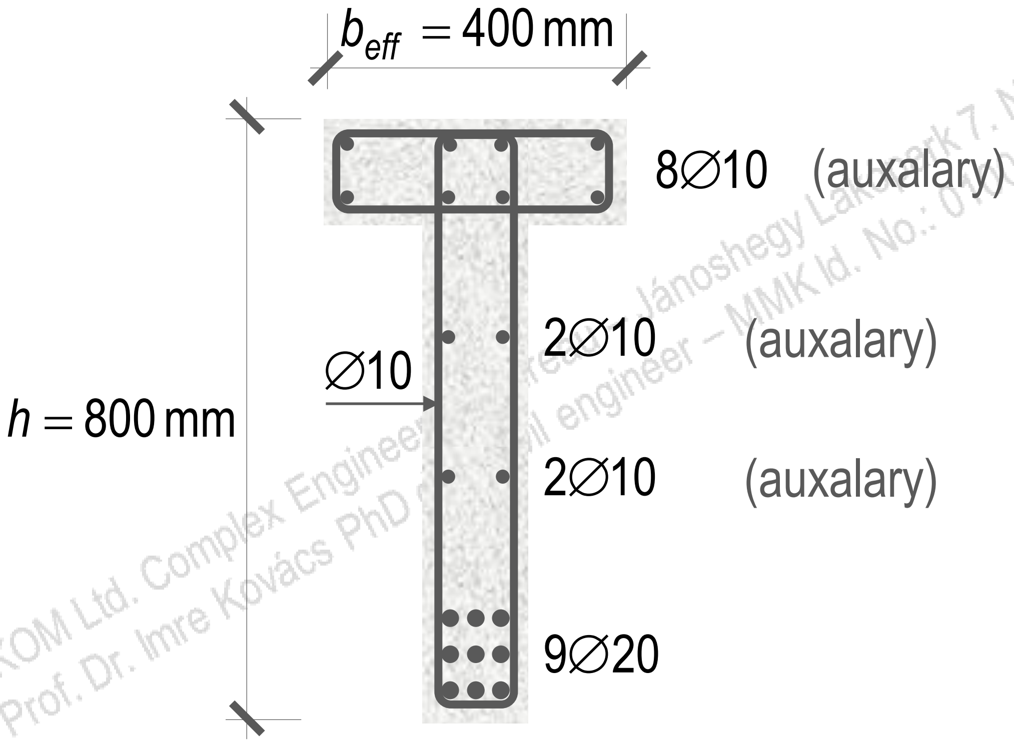
$\varepsilon_t = \dfrac{f_{ctd}}{E_{cd}} = \dfrac{1{,}61}{9500} = 0{,}169\,‰$

$\sigma_t = f_{ctd} = 1{,}47\ \text{N/mm}^2$

$\kappa_I = 0{,}546 \cdot 10^{-6}\ \text{1/mm}$

Example: Analysis of the RC cross-section in the uncracked state of stress (3)

Determine the cracking moment of the outlined prefabricated RC ”T” cross-section (**h = 800 mm, b_{eff} = 400 mm, b_w = 160 mm, v = 160 mm**) in accidental design situation, if the concrete grade is **C50/60**, the nominal concrete cover on stirrups is **C_{nom} = 20 mm**, the maximum size of the aggregates is **d_g = 8 mm**, the diamater of the stirrup reinforcement is **$Ø_s$ = 10 mm**, the tensioned reinforcement consists of **9Ø20,** the auxalary (compressed side) reinforcement consists of **8Ø10, B500B**! Determine and outline the strain and stress distribution of the cross-section at the moment of concrete cracking!



- Concrete grade: **C50/60**
- Aggregate: **d_g = 8 mm**
- RE bar grade: **B500B**
- Height: **h = 800 mm**
- Width of flange: **b_{eff} = 400 mm**
- Width of web: **b_w = 160 mm**
- Thickness of flange: **v = 160 mm**
- Tensioned reinforcement: **$A_{s,prov}$ = 9Ø20 (2826 mm^2)**
- Auxalary reinforcement: **$A_s{}'_{,prov}$ = 12Ø10**
- Stirrups: **Ø10**

Example: Analysis of the RC cross-section in the uncracked state of stress (3)

- $$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{50}{1{,}20} = 41{,}66\ \text{N/mm}^2$$ ☑
  - $$f_{ctm} = 0{,}30 \cdot f_{ck}^{(2/3)} = 0{,}30 \cdot 50^{(2/3)} = 4{,}10\ \text{N/mm}^2$$ ☑
  - $$f_{ctk0{,}05} = 0{,}70 \cdot f_{ctm} = 0{,}70 \cdot 4{,}10 = 2{,}90\ \text{N/mm}^2$$ ☑
  - $$f_{ctk,fl0{,}05} = \max\left\{\begin{matrix}\left(1{,}6 - \dfrac{h}{1000}\right) \cdot f_{ctk0{,}05} \\ f_{ctk0{,}05}\end{matrix}\right\} = \max\left\{\begin{matrix}\left(1{,}6 - \dfrac{800}{1000}\right) \cdot 2{,}90 = 2{,}32\ \text{N/mm}^2 \\ 2{,}90\ \text{N/mm}^2\end{matrix}\right\} = 2{,}90\ \text{N/mm}^2$$ ☑
- $$f_{ctd} = \alpha_{ct} \cdot \frac{f_{ctk,fl0{,}05}}{\gamma_C} = 1{,}00 \cdot \frac{2{,}90}{1{,}20} = 2{,}42\ \text{N/mm}^2$$ ☑
- $$E_{cd} = \frac{f_{cd}}{1{,}75\ {}^0\!/_{00}} = \frac{41{,}66}{1{,}75} \approx 23800\ \text{N/mm}^2$$ ☑
- $$f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}00} = 500\ \text{N/mm}^2$$ ☑
- $$\alpha = E_s / E_{cd} = 200000 / 23800 \approx 8{,}40$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (3)

- $$\Delta\varnothing_{min} = \max\left\{\begin{matrix} k_1 \cdot \varnothing = 1 \cdot 20 = 20\text{ mm} \\ d_g + k_2 = 8 + 5 = 13\text{ mm} \\ 20\text{ mm} \end{matrix}\right\} = 20\text{ mm}$$ ☑

- $$\Delta\varnothing = \frac{b_w - (2 \cdot C_{nom} + 2 \cdot \varnothing_s + n \cdot \varnothing)}{n-1} = \frac{160 - (2 \cdot 20 + 2 \cdot 10 + 3 \cdot 20)}{3-1} = 20\text{ mm} = \Delta\varnothing_{min} = 20\text{ mm}$$ ☑

- $$d = h - \left(C_{nom} + \varnothing_s + \varnothing + \Delta_{sor} + \frac{\varnothing}{2}\right) = 800 - \left(20 + 10 + 20 + 20 + \frac{20}{2}\right) = 720\text{ mm}$$ ☑

➡ $$A_I = b_w \cdot (h - v) + b_{eff} \cdot v + (\alpha - 1) \cdot A_{s,prov} = 160 \cdot (800 - 160) + 400 \cdot 160 + (8{,}40 - 1) \cdot 2826 = 0{,}187 \cdot 10^6\text{ mm}^2$$ ☑

- $$S_x' = b_{eff} \cdot v \cdot \frac{v}{2} + b_w \cdot (h - v) \cdot \left(v + \frac{h - v}{2}\right) + (\alpha - 1) \cdot A_{s,prov} \cdot d$$ ☑
  $$= 400 \cdot 160 \cdot \frac{160}{2} + 160 \cdot (800 - 160) \cdot \left(160 + \frac{800 - 160}{2}\right) + (8{,}40 - 1) \cdot 2826 \cdot 720 = 69{,}30 \cdot 10^6\text{ mm}^3$$

➡ $$x_I = \frac{S_x'}{A_I} = \frac{69{,}30 \cdot 10^6}{0{,}187 \cdot 10^6} = 370\text{ mm}$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (3)

$$I_I = \frac{b_{eff} \cdot v^3}{12} + b_{eff} \cdot v \cdot \left(\frac{v}{2} - x_I\right)^2 + \frac{(h-v)^3 \cdot b_w}{12} + (h-v) \cdot b_w \cdot \left(v + \frac{h-v}{2} - x_I\right)^2 + (\alpha - 1) \cdot A_{s,prov} \cdot (d - x_I)^2 =$$

$$= \frac{400 \cdot 160^3}{12} + 400 \cdot 160 \cdot \left(\frac{160}{2} - 370\right)^2 +$$

$$+ \frac{(800-160)^3 \cdot 160}{12} + (800-160) \cdot 160 \cdot \left(160 + \frac{800-160}{2} - 370\right)^2 +$$

$$+ (8{,}40 - 1) \cdot 2826 \cdot (720 - 370)^2 = 12{,}80 \cdot 10^9 \text{ mm}^4$$ ☑

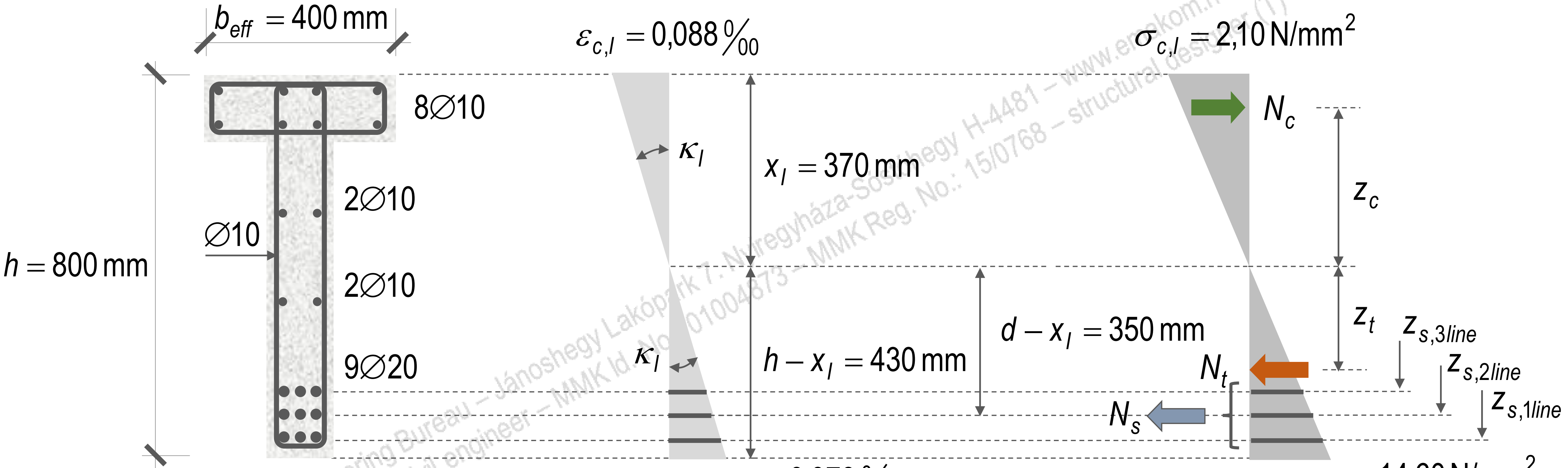
$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I} = 2{,}42 \cdot \frac{12{,}80 \cdot 10^9}{800 - 370} = 72 \text{ kNm}$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (3)

- $\sigma_{c,I} = \frac{M_{crack}}{I_I} \cdot x_I = \frac{72 \cdot 10^6}{12{,}80 \cdot 10^9} \cdot 370 = 2{,}10\,\text{N/mm}^2$ ☑
- $\varepsilon_{c,I} = \frac{\sigma_{c,I}}{E_{cd}} = \frac{2{,}10}{23800} = 0{,}088\,‰$ ☑
- $\sigma_{s,I} = \sigma_{s,I,2line} = \alpha \cdot \frac{M_{crack}}{I_I} \cdot (d - x_I) = 8{,}40 \cdot \frac{72 \cdot 10^6}{12{,}80 \cdot 10^9} \cdot (720 - 370) = 16{,}50\,\text{N/mm}^2$ ☑
- $\varepsilon_{s,I} = \varepsilon_{s,I,2line} = \frac{\sigma_{s,I}}{E_s} = \frac{16{,}50}{200000} = 0{,}083\,‰$ ☑
  - $\sigma_{s,I,1line} = \alpha \cdot \frac{M_{crack}}{I_I} \cdot (d + \varnothing + \Delta_{sor} - x_I) = 8{,}40 \cdot \frac{72 \cdot 10^6}{12{,}80 \cdot 10^9} \cdot (720 + 40 - 370) = 18{,}40\,\text{N/mm}^2$ ☑
  - $\varepsilon_{s,I,1line} = \frac{\sigma_{s,I,1line}}{E_s} = \frac{18{,}40}{200000} = 0{,}092\,‰$ ☑
  - $\sigma_{s,I,3line} = \alpha \cdot \frac{M_{crack}}{I_I} \cdot (d - \varnothing - \Delta_{sor} - x_I) = 8{,}40 \cdot \frac{72 \cdot 10^6}{12{,}80 \cdot 10^9} \cdot (720 - 40 - 370) = 14{,}60\,\text{N/mm}^2$ ☑
  - $\varepsilon_{s,I,3line} = \frac{\sigma_{s,I,3line}}{E_s} = \frac{14{,}60}{200000} = 0{,}073\,‰$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (3)

Geometry	Deformation, ε	Normal stress, σ



$$\varepsilon_{s,I,3line} = 0{,}073\,‰ \qquad \sigma_{s,I,3line} = 14{,}60\,\mathrm{N/mm^2}$$

$$\varepsilon_{s,I,2line} = 0{,}083\,‰ \qquad \sigma_{s,I,2line} = 16{,}50\,\mathrm{N/mm^2}$$

$$\varepsilon_{s,I,1line} = 0{,}092\,‰ \qquad \sigma_{s,I,1line} = 18{,}40\,\mathrm{N/mm^2}$$

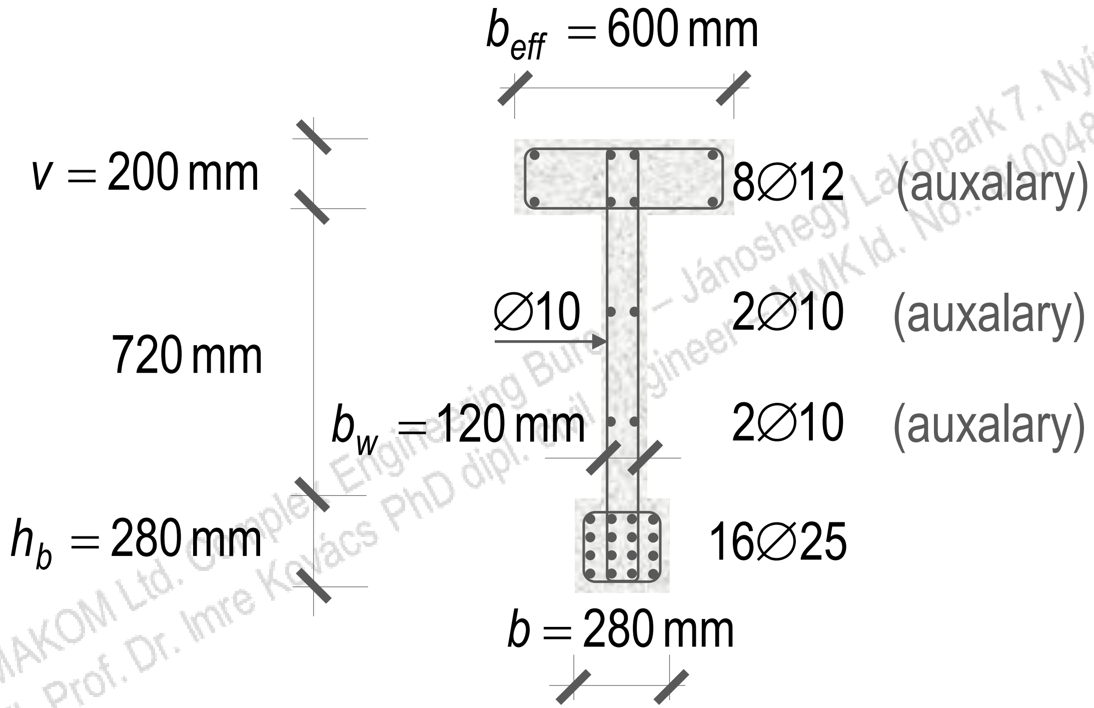
$$\varepsilon_t = f_{ctd} / E_{cd} = 2{,}42/23800 = 0{,}102\,‰ \qquad \sigma_t = f_{ctd} = 1{,}47\,\mathrm{N/mm^2}$$

$$\kappa_I = \frac{\varepsilon_{c,I}}{x_I} = \frac{0{,}088}{370} = 0{,}239 \cdot 10^{-6}\ \tfrac{1}{\mathrm{mm}}$$

$$M_I = M_{crack} = 72\,\mathrm{kNm}$$

Example: Analysis of the RC cross-section in the uncracked state of stress (4)

Determine the cracking moment of the outlined prefabricated RC ”I” cross-section (**h = 1200 mm, b_{eff} = 600 mm, b_w = 120 mm, v = 200 mm, b = 280 mm, h_b = 280 mm**) in seismic design situation, if the concrete grade is **C40/50**, the nominal concrete cover on stirrups is **C_{nom} = 20 mm**, the maximum size of the aggregates is **d_g = 8 mm**, the diamater of the stirrup reinforcement is **$Ø_s$ = 10 mm**, the tensioned reinforcement consists of **16Ø25,** the auxalary (compressed side) reinforcement consists of **4Ø10+ 8Ø12, B500B**! Determine and outline the strain and stress distribution of the cross-section at the moment of concrete cracking!



- Concrete grade: **C40/50**
- Aggregate: **d_g = 8 mm**
- Re bar grade: **B500B**
- Height: **h = 1200 mm**
- Width of top flange: **b_{eff} = 600 mm**
- Width of web: **b_w = 120 mm**
- Thickness of top flange: **v = 200 mm**
- Width of bottom flange: **b = 280 mm**
- Thickness of bottom flange: **h_b = 280 mm**
- Main reinforcement **$A_{s,prov}$ = 16Ø25 (7856 mm^2)**
- Auxalary reinforcement: **$A_{s',prov}$ = 4Ø10+8Ø12**
- Stirrups: **Ø10**

Example: Analysis of the RC cross-section in the uncracked state of stress (4)

$$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} = 1{,}00 \cdot \frac{40}{1{,}20} = 33{,}33\,\text{N/mm}^2$$ ☑

- $$f_{ctm} = 0{,}30 \cdot f_{ck}^{(2/3)} = 0{,}30 \cdot 40^{(2/3)} = 3{,}50\,\text{N/mm}^2$$ ☑
- $$f_{ctk\,0{,}05} = 0{,}70 \cdot f_{ctm} = 0{,}70 \cdot 3{,}50 = 2{,}45\,\text{N/mm}^2$$ ☑
- $$f_{ctk,fl\,0{,}05} = \max\left\{\begin{matrix}\left(1{,}6 - \frac{h}{1000}\right) \cdot f_{ctk\,0{,}05} \\ f_{ctk\,0{,}05}\end{matrix}\right\} = \max\left\{\begin{matrix}\left(1{,}6 - \frac{1200}{1000}\right) \cdot 2{,}45 = 0{,}98\,\text{N/mm}^2 \\ 2{,}45\,\text{N/mm}^2\end{matrix}\right\} = 2{,}45\,\text{N/mm}^2$$ ☑

$$f_{ctd} = \alpha_{ct} \cdot \frac{f_{ctk,fl\,0{,}05}}{\gamma_C} = 1{,}00 \cdot \frac{2{,}45}{1{,}20} = 2{,}00\,\text{N/mm}^2$$ ☑

$$E_{cd} = \frac{f_{cd}}{1{,}75\,‰} = \frac{33{,}33}{1{,}75} \approx 19000\,\text{N/mm}^2$$ ☑

$$f_{yd} = \frac{f_{yk}}{\gamma_S} = \frac{500}{1{,}00} = 500\,\text{N/mm}^2$$ ☑

$$\alpha = E_s / E_{cd} = 200000 / 19000 \approx 10{,}50$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (4)

- $$\Delta\varnothing_{min} = \max\begin{Bmatrix} k_1 \cdot \varnothing = 1 \cdot 25 = 25\ \text{mm} \\ d_g + k_2 = 8 + 5 = 13\ \text{mm} \\ 20\ \text{mm} \end{Bmatrix} = 25\ \text{mm}$$ ☑
- $$\Delta\varnothing = \frac{b - (2 \cdot C_{nom} + 2 \cdot \varnothing_s + n \cdot \varnothing)}{n-1} = \frac{280 - (2 \cdot 20 + 2 \cdot 10 + 4 \cdot 25)}{4-1} = 40\ \text{mm} > \Delta\varnothing_{min} = 25\ \text{mm}$$ ☑
- $$d = h - \left(C_{nom} + \varnothing_s + \varnothing + \Delta_{sor} + \varnothing + \frac{\Delta_{sor}}{2}\right) = 1200 - \left(20 + 10 + 20 + 40 + 20 + \frac{40}{2}\right) = 1070\ \text{mm}$$ ☑

➡ $$A_I = b \cdot h_b + b_{eff} \cdot v + b_w \cdot (h - v - h_b) + (\alpha - 1) \cdot A_{s,prov} =$$ ☑

$$= 280 \cdot 280 + 600 \cdot 200 + 120 \cdot (1200 - 200 - 280) + (10{,}50 - 1) \cdot 7856 = 0{,}359 \cdot 10^6\ \text{mm}^2$$

Example: Analysis of the RC cross-section in the uncracked state of stress (4)

- $$S_x' = b_{eff} \cdot v \cdot \frac{v}{2} + (b \cdot h_b) \cdot \left(h - \frac{h_b}{2}\right) + b_w \cdot (h - v - h_b) \cdot \left(v + \frac{h - v - h_b}{2}\right) + (\alpha - 1) \cdot A_{s,prov} \cdot d =$$

$$= 600 \cdot 200 \cdot \frac{200}{2} + (280 \cdot 280) \cdot \left(1200 - \frac{280}{2}\right) + 120 \cdot (1200 - 200 - 280) \cdot \left(200 + \frac{1200 - 200 - 280}{2}\right) +$$

$$+ (10{,}5 - 1) \cdot 7856 \cdot 1070 = 0{,}223 \cdot 10^9 \text{ mm}^3$$ ☑

➡ $$x_I = \frac{S_x'}{A_I} = \frac{0{,}223 \cdot 10^9}{0{,}359 \cdot 10^6} = 621 \text{mm}$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (4)

$$I_I = \frac{b_{eff} \cdot v^3}{12} + b_{eff} \cdot v \cdot \left(\frac{v}{2} - x_I\right)^2 + \frac{h_b{}^3 \cdot b}{12} + b \cdot h_b \cdot \left(h - \frac{h_b}{2} - x_I\right)^2$$

$$\frac{(h - v - h_b)^3 \cdot b_w}{12} + (h - v - h_b) \cdot b_w \cdot \left(v + \frac{h - v - h_b}{2} - x_I\right)^2 + (\alpha - 1) \cdot A_{s,prov} \cdot (d - x_I)^2 =$$

$$= \frac{600 \cdot 200^3}{12} + 600 \cdot 200 \cdot \left(\frac{200}{2} - 621\right)^2 + \frac{280^3 \cdot 280}{12} + 280 \cdot 280 \cdot \left(1200 - \frac{280}{2} - 621\right)^2 +$$

$$+ \frac{(1200 - 200 - 280)^3 \cdot 120}{12} + (1200 - 200 - 280) \cdot 120 \cdot \left(200 + \frac{1200 - 200 - 280}{2} - 621\right)^2 +$$

$$+ (10{,}5 - 1) \cdot 7856 \cdot (1070 - 621)^2 = 52{,}60 \cdot 10^9 \text{ mm}^4$$ ☑

$$M_I = M_{crack} = f_{ctd} \cdot \frac{I_I}{h - x_I} = 2{,}00 \cdot \frac{52{,}60 \cdot 10^9}{1200 - 621} = 182 \text{ kNm}$$ ☑

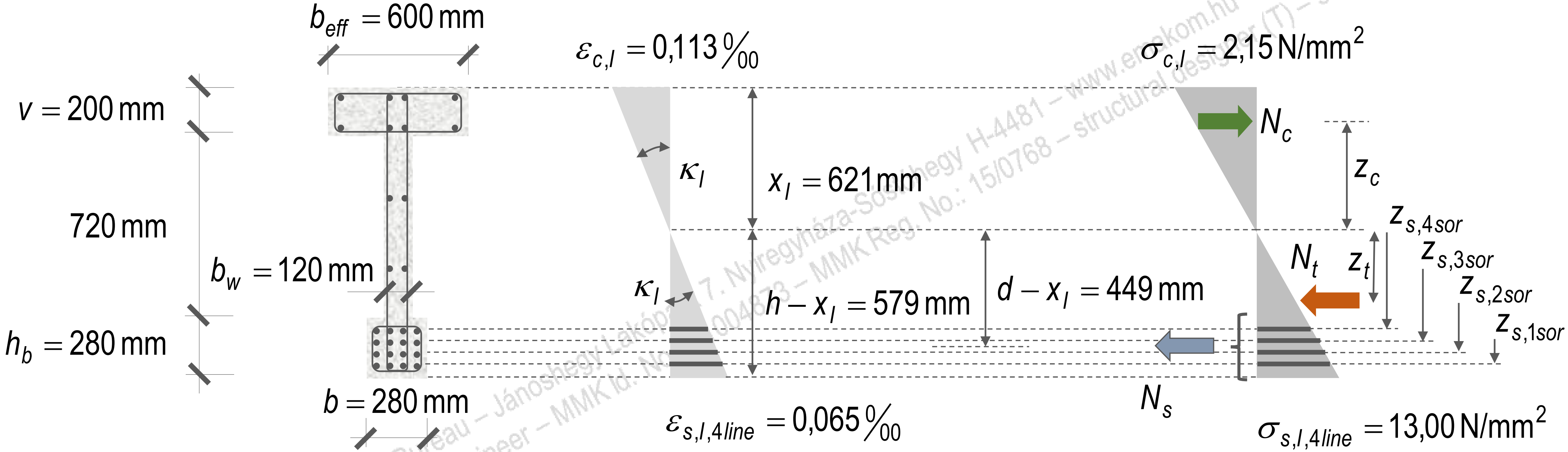
Example: Analysis of the RC cross-section in the uncracked state of stress (4)

- $$\sigma_{c,I} = \frac{M_{crack}}{I_I} \cdot x_I = \frac{182 \cdot 10^6}{52{,}60 \cdot 10^9} \cdot 621 = 2{,}15\,\text{N/mm}^2$$ ☑
- $$\varepsilon_{c,I} = \frac{\sigma_{c,I}}{E_{cd}} = \frac{2{,}15}{19000} = 0{,}113\,‰$$ ☑
- $$\sigma_{s,I} = \alpha \cdot \frac{M_{crack}}{I_I} \cdot (d - x_I) = 10{,}50 \cdot \frac{182 \cdot 10^6}{52{,}60 \cdot 10^9} \cdot (1070 - 621) = 16{,}30\,\text{N/mm}^2$$ ☑
- $$\varepsilon_{s,I} = \frac{\sigma_{s,I}}{E_s} = \frac{16{,}31}{200000} = 0{,}082\,‰$$ ☑

- $$\left\{\begin{array}{ll} \sigma_{s,I,4line} = 13{,}00\,\text{N/mm}^2 & \varepsilon_{s,I,4line} = 0{,}065\,‰ \\ \sigma_{s,I,3line} = 15{,}20\,\text{N/mm}^2 & \varepsilon_{s,I,3line} = 0{,}076\,‰ \\ \sigma_{s,I,2line} = 17{,}40\,\text{N/mm}^2 & \varepsilon_{s,I,2line} = 0{,}087\,‰ \\ \sigma_{s,I,1line} = 19{,}60\,\text{N/mm}^2 & \varepsilon_{s,I,1line} = 0{,}098\,‰ \end{array}\right\}$$ ☑

Example: Analysis of the RC cross-section in the uncracked state of stress (4)

Geometry	Deformation, ε	Normal stress, σ



$\varepsilon_{s,I,4line} = 0{,}065\,‰$ $\qquad \sigma_{s,I,4line} = 13{,}00\,\text{N/mm}^2$

$\varepsilon_{s,I,3line} = 0{,}076\,‰$ $\qquad \sigma_{s,I,3line} = 15{,}20\,\text{N/mm}^2$

$\varepsilon_{s,I,2line} = 0{,}087\,‰$ $\qquad \sigma_{s,I,2line} = 17{,}40\,\text{N/mm}^2$

$\varepsilon_{s,I,1line} = 0{,}098\,‰$ $\qquad \sigma_{s,I,1line} = 19{,}60\,\text{N/mm}^2$

$$\kappa_I = \frac{\varepsilon_{c,I}}{x_I} = \frac{0{,}113}{621} = 0{,}182 \cdot 10^{-6}\ {}^{1}\!/_{\text{mm}}$$

$$M_I = M_{crack} = 182\,\text{kNm}$$

$$\varepsilon_t = f_{ctd} / E_{cd} = 2{,}00/19000 = 0{,}105\,‰$$

$$\sigma_t = f_{ctd} = 2{,}00\,\text{N/mm}^2$$

Reinforced Concrete (RC) Structures

Topic 13.

Uncracked state of stress - I. state of stress

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Thank you for your kind attantion!